

The marginal cost of justice: A theory of optimal use of alternative criminal procedures

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(Preliminary draft, please do not cite.)

Abstract

Criminal cases can be adjudicated via court trials or alternative criminal procedures, such as penal order and plea bargaining. We develop a model of optimal allocation of cases across these alternatives. The model predicts that the evidence standards—and thus the number of wrongful convictions and wrongful acquittals—fundamentally depend upon the cost structure of the criminal process as well as on the budget resources allocated to the criminal justice system. We call this phenomenon the marginal cost of justice. Our model offers explanations why the scope of plea bargaining in the United States has been traditionally broader than the scope of alternative procedures in Europe and why their use in Europe has recently expanded.

JEL classification: K14, K41, K42.

Keywords: Criminal procedure, law enforcement, legal process.

1 Introduction

1.1 Motivation

Criminal justice has two goals: acquittal of the innocent and conviction of the guilty—in that order. However, the difficulty of finding out who is innocent and who is guilty varies across cases: evidence may be fuzzy and costly to obtain; if this wasn't the case, courts would need not exist. Courts have emerged as a prime mechanism through which societies strive to solve the problem of criminal justice and to avoid the two possible errors: convicting the innocent and releasing the guilty.

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At the same time, resources are constrained and trials are a very costly way to make decisions. Constraints result in compromises. In particular, not every case is adjudicated by a court. At one extreme, cases are dropped by state attorneys if the evidence is weak. At the other extreme, strong cases, but typically minor offenses, are decided through alternative, less-than-trial procedures. The police, for instance, usually has the authority to issue a parking ticket and unless the driver appeals, this decision is enforceable. Although the police cannot adjudicate more serious offenses, it typically has the authority to drop the case if the evidence is weak. Lesser offenses carrying short jail sentence or non-jail sentences can sometimes be decided through accelerated or simplified procedures that are common in Europe. In the United States, most crimes are resolved via plea bargain and they never reach the trial. Why do such procedures exist? Why have they become more common, particularly in Europe? What is the socially optimal standard determining the eligibility of a case to be resolved without trial? What are the effects of budgetary limits and shifts in absolute or relative costs associated with each procedure?

1.2 Institutional background

The economic analysis of criminal procedure has focused predominantly on plea bargaining in its U.S. institutional setting. However, most European countries also use alternatives to the standard trial. Although the institutional details vary by country, below we summarize their key common features and give examples of countries in which they are used.¹

- *Penal order* (e.g., Germany, the Netherlands, Sweden, Czech Republic) The judge, upon reviewing the prosecution, may issue a penal order, in which the defendant is convicted and the sentence is set. The defendant may appeal the penal order, in which case the case reverts to trial. Only a limited range of sanctions can be imposed through the penal order (monetary fine, parole, suspension of the driving license, etc.); prison sentences can be imposed only in some countries and if so, under certain conditions and up to a relatively short length.

¹The description is based on Zeman et al. (2013), p. 14-38.

- *Accelerated/simplified proceedings* (e.g., Great Britain, Spain, the Netherlands, Czech Republic, Slovakia). Their purpose is to resolve evidentiarily simple cases quickly and with less paperwork than in a standard trial. The formal requirements for the justification of the prosecution and the judicial verdict are simpler, some of the steps involving the presentation of evidence may be skipped, the case is adjudicated by a single judge (e.g., the so-called police judge in the Netherlands), and the law may set deadlines. The accelerated/simplified proceedings are restricted to less severe crimes and to cases where the evidence is sufficiently clear (e.g., the offender was arrested on the crime scene).
- *Plea bargaining* (e.g., Poland, Slovakia). The defendant may confess to the charges raised by the prosecutor and accept the proposed sentence. That way, he avoids a costly trial and a potentially more severe sentence that could be imposed if he were convicted at trial. The European implementations of plea bargaining differ from their American predecessor. The legislation sets the maximum “discount” from the sentence that the defendant can obtain by accepting the plea bargain. The judge has to approve the plea bargain and a non-negligible fraction of the bargains is rejected.
- Procedures combining the above features. For example, France’s *la comparution sur reconnaissance préalable de culpabilité* has the essential feature of plea bargaining (the defendant’s consent) but with a more active judicial involvement. Or the Netherlands’ *transactie*, in which the defendant can pay the proposed fine and thus avoid further prosecution, though the payment of the fine does not imply confession.

The alternative procedures proliferated throughout Europe in the last two decades. Table 1 shows list of countries that use each of the three broader types of alternative procedures. While the table is admittedly incomplete, it shows several broad trends. The alternative procedures were intensively adopted during 2000s. Several countries use two or three types of alternative procedures. The scope of the procedures is typically limited, both by limiting the offenses that are eligible for the procedure, and by limiting the punishment that can be imposed. The penal order is very limited in terms of the punishment - the prison sentence is either not allowed, or it is limited to a suspended sentence. That effectively limits the use of the penal order to minor crimes, unless such use is also limited explicitly by law. The accelerated/simplified proceedings

Table 1: Alternative criminal procedures in selected European countries

	Introduced	Eligible crimes	Constraints on punishment
<i>Penal Order</i>			
Czech Republic	1994	All	Up to one year suspended prison sentence
France	2002	Very minor crimes	Prison sentence cannot be imposed
Germany	1877	Minor crimes	Up to one year suspended prison sentence
Hungary	TBA	Crimes with a three-year maximum statutory sentence	Limited range of punishments
Netherlands	2008		Prison sentence cannot be imposed
Poland	TBA		Prison sentence cannot be imposed
Sweden	TBA	Minor crimes	Up to a suspended prison sentence
<i>Accelerated/Simplified Proceedings</i>			
Czech Republic	2002	Factually clear crimes with a five-year maximum statutory sentence	Limited by the eligible crime
Germany	TBA	Factually clear crimes	Up to one year prison sentence
Hungary	TBA	Factually clear crimes with a eight-year maximum statutory sentence	Limited by the eligible crime
Poland	2007	Selected minor crimes	Limited by the eligible crime
Slovakia	2002	Crimes with a five-year maximum statutory sentence	Limited by the eligible crime
Spain	1988	<i>Juicio de faltas</i> : crimes not punishable by imprisonment	Limited by the eligible crime
Spain	2002	<i>Juicio rapidos</i> : selected, factually clear crimes with a five-year maximum statutory sentence	Limited by the eligible crime
<i>Plea Bargain</i>			
Czech Republic	2012	Crimes with a ten-year maximum statutory sentence	Limited by the eligible crime
France	2004	Crimes with a five-year maximum statutory sentence	Up to one year prison sentence or one half of the statutory sentence
Hungary	TBA	Crimes with a eight-year maximum statutory sentence	Limited by the eligible crime
Poland	2003	Factually clear crimes with a ten-year maximum statutory sentence	Limited by the eligible crime
Slovakia	2005	All crimes	None

are typically limited to minor crimes (frequently defined by the maximum statutory sentence) and to crimes that are factually simple, with little doubt about the defendant's guilt. The definition of eligible offenses in turn limits the punishment, and in practice more severe punishments can be imposed through the accelerated/simplified proceedings than through the penal order. Last, plea bargaining is least restrictive in terms of the eligible crimes.

These alternative criminal procedures share the same underlying economic logic. They are cheaper than the conventional trials. They require far less time input on behalf of the judges, prosecutors, defendants, and their attorneys than the trial. They have positive costs, however: Negotiating the guilty plea or issuing the penal order is more time consuming for the prosecutor and the judge than dropping the charges altogether. They lead to a conviction of the defendant with a probability one or close to one, far higher than the probability of conviction at trial. They generally require a lower amount of evidence and examination of the evidence by the judge. Our model captures this common economic logic while intentionally abstracting from the finer institutional differences between alternative procedures and countries.

1.3 Contribution of this paper

We develop a general theory of criminal process in order to understand the allocation of cases across alternative procedures. The normative model represents the prosecutors and judges as one benevolent agent labelled "adjudicator". The adjudicator has a population of cases to be decided. Her objective is to minimize the sum of the cost of wrongful convictions, wrongful acquittals and the criminal justice process itself. The total costs of the criminal justice process itself are limited by the resource constraint. The available choices are to drop the case, convict the defendant through the administrative procedure (without trial) based on the available evidence, or conduct a costly trial in which more evidence will be available. The cases differ by offense severity (high or low) and by the strength of the available evidence p . We derive the optimal decision rule. It consists of thresholds of evidence p_T and p_C such that cases with evidence below p_T are dropped, with evidence between p_T and p_C are decided at trial and with evidence above p_C are convicted without trial. We investigate how the optimal rule depends on the offense

severity, cost of trial, the distribution of the evidence in the entire population of cases, and the resource constraint.

The model yields some intuitive and some non-intuitive results. More severe offenses require higher standard of evidence for conviction without trial and lower standard for dropping the cases; hence trials are more prevalent among more severe offenses. However, it is always optimal to allow the conviction without trial even for the most severe crimes. A tightening of the resource constraint reduces the fraction of trials and increases the total costs of errors. The model explicitly states the “marginal cost of justice”: it is equal to the marginal cost of wrongful convictions and acquittals due to reducing the criminal procedure budget by one dollar. A change in any parameter affects the optimal thresholds p_T and p_C for both levels of offense severity through its effect on the resource constraint.

The model can explain, for example, why plea bargaining in the United States is used even for the most severe offenses while the alternative procedures in Europe are generally restricted to less severe offenses. It also offers explanations for the proliferation of simplified, alternative criminal procedures in Europe in recent decades. One explanation is that more binding resource constraints forced the adoption of cheaper alternatives. The other, perhaps paradoxical explanation, is that the alternative procedures were adopted for low-severity crimes because the societies became *more* concerned with the cost of wrongful convictions and wrongful acquittals.

This paper makes a contribution to several strands of the literature. First, it develops formally the classical Law and Economics view of plea bargaining. The early literature (Easterbrook 1983, Friedman 2000, ch. 4) has postulated the “resource releasing” defense. Resolving cases through plea bargaining (a low-cost procedure) releases resources that the prosecutor can allocate into prosecuting the difficult cases more vigorously. The probability of conviction is higher as a consequence. The only formal model capturing the resource releasing argument is by Givati (2014). He models the decision of the benevolent government whether to allow plea bargaining or not, based on the available resource constraint, the probabilities of judicial errors, and the underlying choice of defendants of accepting or rejecting the plea offer based on their risk aversion. He finds that the decision to adopt plea bargaining is inversely related to the society’s aversion to judicial errors and to the fraction of factually innocent defendants in the population of

arrests. Our model yields additional subtle predictions, such as that allowing plea bargaining (or any other alternative procedure) for low-severity cases also reduces the fraction of high-severity cases that are dropped and reduces the total sum of the cost of errors, despite an increase in the cost of errors for the subset of cases that are resolved through plea bargaining. More importantly, we generalize the “resource-releasing” hypothesis to a wider range of alternative procedures and highlight their common equilibrium effects.

Second, this paper builds upon the literature on the optimal standard of conviction (Andreoni 1991, Doménech and Puchades 2014, Lando 2009, Rizzolli and Saraceno 2011). In these papers, the adjudicator who issues the verdict compares the expected costs of errors and other social costs of each decision, and the standard of conviction is determined endogenously as a function of the characteristics of a particular case. Most closely to our question of interest, Doménech and Puchades’ (2014) model the choice after arrest, where the available option is to drop the case or proceed to a costly trial, in which precise evidence will become available. We add the alternative procedure into this framework - that is, the possibility to convict the defendant at low cost without trial, and derive the standards for such conviction. We show that the standard depends also on the distribution of case characteristics in the entire population of cases and on the available resources.

Last, several authors modeled the plea bargaining as a game between the game and the prosecutor (Grossman and Katz 1983, Miceli and Adelstein 2001, Mongrain and Roberts 2009, Reinganum 1988). The general tendency in these models is that the factually guilty defendants are far more likely to accept the plea bargain than the factually innocent (some models predict perfect sorting). In a sense, the plea bargaining generates an additional signal of evidence of guilt. Our model provides a simple normative standard as to which cases should be resolved through plea bargaining while not explicitly modeling the plea bargaining tactics. And, given that plea bargaining is available as the main alternative to trial, the model explains why more severe can be resolved through plea bargaining while the other (European) alternatives may optimally be restricted to less severe offenses.

2 Model assumptions

A benevolent adjudicator is tasked with deciding criminal cases. Her objective is to minimize the social costs of the criminal process. The social costs are composed of the social costs of wrongful acquittals, w_a , which includes costs of weaker deterrence and non-incapacitation, the social costs of wrongful convictions, w_c , which includes the costs born by the unjustly convicted person and the cost to society of letting the true offender loose, and the costs of the decision-making process. Most plausibly the cost of wrongful conviction exceed the cost of wrongful acquittal, $w_c > w_a$.² However, the results do not qualitatively hinge on this assumption, therefore we do not explicitly impose it. There are three ways in which the adjudicator processes a case: (i) it can be dropped, with zero cost, (ii) it can result in a conviction without a trial, which costs c_C , or (iii) it can be passed to a court, which costs c_T . We assume $c_T > c_C > 0$. Intuitively, for a policeman tasked to give out parking tickets, for example, the cost of passing an opportunity to issue a ticket is zero, whereas issuing a ticket costs the effort of actually writing the ticket and collecting the documentation for that particular offense. Negotiating a plea bargain takes more time than unilaterally dropping the charges.

Initially, there is an uncertainty about the guilt of the defendant, that is the available evidence is imperfect. Let p denote the probability of the defendant's guilt, which the adjudicator infers from the evidence available to her.³

²The assumption that $w_c > w_a$ is common to many models of criminal procedure and dates back to Andreoni (1991). Rizzolli and Saraceno (2011) show that the costs of wrongful convictions are indeed greater than the costs of wrongful acquittals through their asymmetric effects on deterrence and the cost of punishment.

³The inferred probability of guilt can be rationalized with the probabilistic deduction structure (Sanchirico 2012). In the population of arrested defendants, the probability that a randomly selected defendant is guilty is $p(G)$. The evidence about the individual defendant's guilt is summarized in a signal $\theta \in [0, 1]$. The signal θ is informative: the factually guilty defendants are more likely to have higher values of θ than the factually innocent. The distribution of evidence for the guilty $F^G(\theta)$ first-order stochastically dominates the distribution for the innocent $F^I(\theta)$, which is a technical definition of an informativeness of the signal. (That is, $F^G(\theta) < F^I(\theta) \forall \theta$.) The distribution of evidence in the entire population of defendants is the pooled distribution of the two distributions: $F(\theta) = p(G) F^G(\theta) + (1 - p(G)) F^I(\theta)$.

The adjudicator knows $p(G)$ and the distribution functions $F^G(\theta)$ and $F^I(\theta)$. She infers the probability of factual guilt from the evidence, as implied by the Bayes' rule

$$p = \frac{l(\theta) p(G)}{l(\theta) p(G) + (1 - p(G))},$$

where $l(\theta) = f^G(\theta) f^I(\theta)$ is the relative density of observing θ for a guilty person and an innocent person, and is increasing in θ . It is a useful summary indicator of informativeness of θ . It is straightforward to verify that $\partial p / \partial \theta > 0$. The results of our model depend only on the values of p and the resulting distribution of p in the population; we therefore do not explicitly model the underlying structure that generates p .

As a result, dropping a case produces costs of wrongful acquittal with probability p , and convicting the defendant without a trial carries a risk of wrongful conviction, with probability $1 - p$.

At trial, new evidence that is more informative of factual guilt or innocence is revealed. For simplicity we assume that the truth is revealed at trial and the case gets decided correctly, implying that there are no social costs associated with trial except c_T . This assumption makes the presentation of our model more lucid without affecting the qualitative results.

3 Benchmark model—unconstrained optimum with one offense type

3.1 Optimal decision rule

For expositional clarity, we first show the key trade-offs in a simplified benchmark model. For now we assume that the adjudicator is not constrained by the budget. Rather, the budget is endogenously determined as the minimum cost of adjudicating the cases that is necessary to reach the cost-minimizing solution. This will allow us to ascertain the conditions under which the social optimum obtains. To simplify the exposition further, there is only one offense type with costs of wrongful acquittal and conviction w_a and w_c .

The adjudicator observes the signal of guilt p for each defendant and decides whether to drop the charges, convict the defendant without trial at low cost, or to conduct a costly trial at which the true guilt or innocence will be ascertained. Given the inferred probability of guilt p , the expected social cost of each decision are:

- Drop the charges: $SC_D = pw_a$. The probability that the released defendant is in fact guilty times the cost of wrongful acquittal.
- Trial: $SC_T = c_T$. The cost of trial itself but no costs of errors.
- Convict without trial: $SC_C = (1 - p)w_c + c_C$. The probability that the convicted defendant is in fact innocent times the cost of wrongful conviction plus the procedural cost of conviction without the trial.

Seeing this trade-off, an unconstrained adjudicator simply chooses the decision with the lowest social cost. Unlike in the full model of subsection 4 she does not need to take into account the cost consequences of her decisions in the other cases.

The choice rule as a function of the inferred probability of guilt is depicted in Figure 1. The cost of trial c_T are assumed to be independent of the inferred probability of guilt. The cost of dropping the case rise linearly in p from zero to w_a and we assume they cross the c_T line at a sufficiently high probability.⁴ The cost of convicting without trial decline linearly from the level of $w_c + c_C$ when the inferred probability of guilt is zero to c_C when the convicted defendant is certain to be guilty. Because by assumption $c_C < c_T$, the cost of convicting without trial must cross the c_T line.⁵

The adjudicator thus follows a simple optimal choice rule: If the signal of guilt is weak enough, the case is dropped. If the signal exceeds a critical level p_T^* , the case goes to trial: the probability of guilt is high enough to justify the cost of conducting the trial and revealing the true guilt. However, if the signal of guilt exceeds the critical level p_C^* , the social cost is minimized if the defendant is convicted without trial. The certainty of convicting the guilty person is high enough to be worth the cost saving associated with the simpler alternative procedure. Formally, the critical levels p_T^* and p_C^* are given by

$$p_T^* w_a = c_T \quad \text{and} \tag{1}$$

$$(1 - p_C^*) w_c + c_C = c_T.$$

Two possible scenarios are shown in Figure 1. In the left panel the evidence threshold at which the social cost of dropping the case are equal to the cost of pursuing the trial, p_T , as well as the threshold where the social cost of conviction without trial is equal to the cost of trial, p_C , depend on the cost of trial. This implies, that the cost of trial essentially determines the optimal standards of evidence for dropping a case or convicting without a trial. This point can be further illustrated in a specific set up when all defendants are either acquitted by the adjudicator or

⁴This assumption is necessary to guarantee the existence of a trial. If the offense is trivial enough, it may still be cheaper from the social point of view to release the defendant than to conduct a costly trial.

⁵Similarly, we assume that $w_c + c_C > c_T$ such that it is socially cheaper to conduct a trial than to convict without trial even the defendants with a very weak signal of guilt.

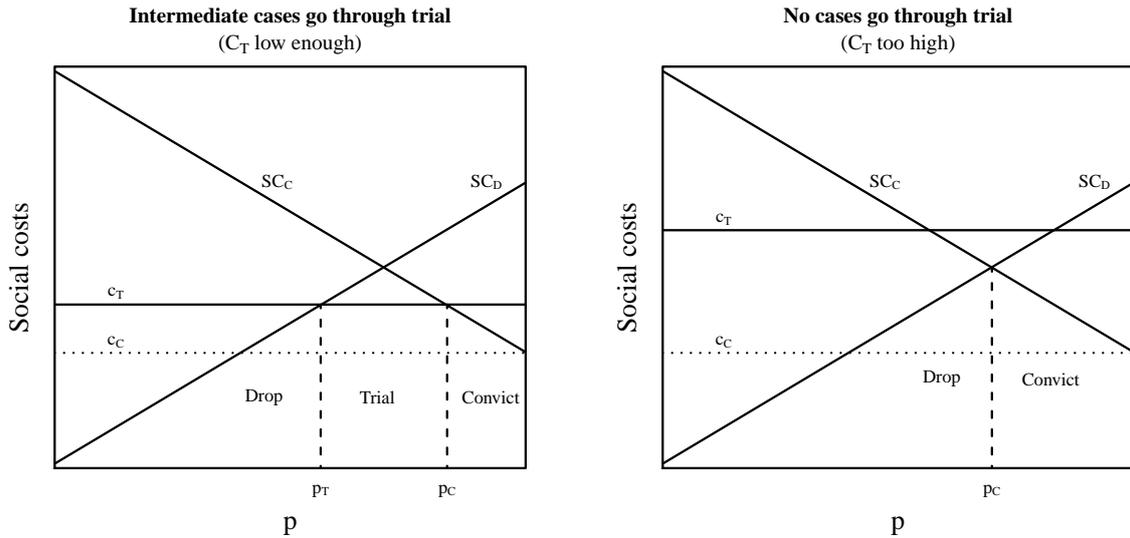


Figure 1: The optimal allocation of cases across available criminal procedures and the evidence standard for dropping the case and conviction without trial depend upon the cost of trial.

convicted, without any case being heard by the court. This situation is shown in the right panel of Figure 1. Intuitively, this result often obtains for small offenses, such as double parking, that are best dealt with administratively.

The observed fraction of cases that are dropped, resolved at trial, or convicted without trial depends not only on the parameters in equation 1 but also on the distribution of the signal p in the population. For example, p_c^* may be high, but if the signals are highly informative, the distribution of p would be highly bimodal. A large fraction of defendants would then be convicted without trial or dropped, and only a small fraction would proceed to trial.

3.2 Implications

Notwithstanding its simplicity, the benchmark model yields several insights that which may help in explaining the empirically observed common structure of the real-world criminal justice systems. We summarize them in the following points:

- If trials were costless, every case would be adjudicated at trial. The very reason why charges are frequently dropped or why the simpler procedures are used at all is that trials are costly. Otherwise more precise verdicts would be reached by giving every case a hearing before trial.

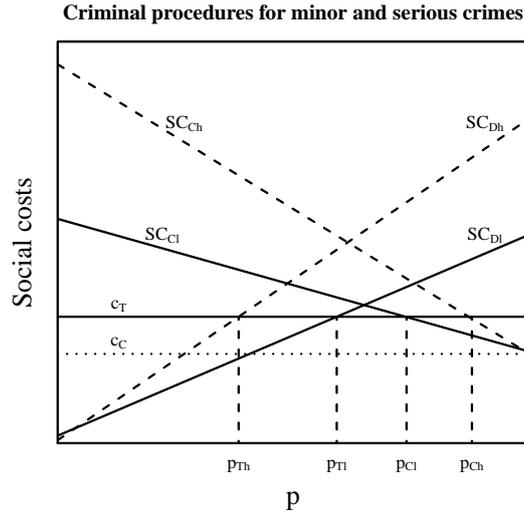


Figure 2: The optimal allocation of cases across available criminal procedures and the evidence standards for dropping the case and conviction without trial depend upon crime severity.

- As the offense severity increases, a higher fraction of defendants is adjudicated at trial and a smaller fraction of defendants is either dropped or convicted without trial. This is apparent from equation (1) and is illustrated in Figure 2. Greater severity of the offense is manifested in this model by higher values of w_a and w_c . Since p is increasing in evidence, an increase in w_a or w_c causes a reduction in p_T^* (fewer cases are dropped) and an increase in p_C^* (fewer cases are convicted without trial). This prediction helps explain why the alternative procedures in the European countries are typically restricted to less severe offenses by law. Similarly, the fraction of defendants accepting the guilty plea (i.e., convicted without trial) is lower in murder cases than in less severe cases.⁶
- As the signals become more informative (the guilty are systematically more likely to draw high signals and vice versa), the fraction of cases that are dropped or convicted without trial increases.
- A decrease in the cost of conviction without trial increases the fraction of defendants convicted without trial, reduces the fraction of defendants at trial, and does not affect the fraction of defendants whose charges are dropped. As a consequence, the overall probability of conviction increases because the marginal defendants who were previously convicted at trial with probability less than one are now convicted without trial with

⁶Source: Authors' own calculation based on State Court Processing Statistic.

certainty. However, the probability of conviction at trial decreases, because the marginal trial cases have the highest probability of conviction at trial.

4 Full model—constrained optimum with two offense types

The social cost in this model includes two types of costs: The costs of erroneous decisions are borne by the defendants and the society at large. The cost of the criminal procedure c_T and c_C (times the number of cases adjudicated through either procedure) are borne by the adjudicator herself. The real-world adjudicators (judges, prosecutors) rarely live in the luxury of being able to reach the cost-minimizing decision in each case, knowing that they will be provided whatever budget is needed to cover the resulting cost of procedure. Rather, a resource constraint is the daily fact of life of law enforcers. The police, prosecutors and judges face—at least in the short run—a fixed amount of time and money and have to choose how to allocate these scarce resources across the entire portfolio of cases. The importance of the resource constraint has been recognized in several models of prosecutor choices and plea bargaining (Landes 1971, Mongrain and Roberts 2009).

Our main model investigates the optimal use of the alternative criminal procedures under the resource constraint. The operating assumption is that the budget available to the adjudicator is smaller than what would be implied by the unconstrained optimum. The constraint implies that the adjudicator has to economize on the use of the costly procedures (most importantly, the trials).

To capture the key trade-offs, we further assume that there are two types of offenses, high-severity and low-severity crimes, with the cost of wrongful acquittal and conviction satisfying $w_{al} < w_{ah}$, $w_{cl} < w_{ch}$. The two types have equal shares. We then also investigate how the presence of the resource constraint affects the relative use of trial or simplified procedure in high-severity and low-severity cases.

4.1 The model and solution

Like in the unconstrained model, the objective of the adjudicator is to choose the decision rule, that is, the critical points p_{Tj} and p_{Cj} that determine whether the case would be dropped, go to

trial, or convicted on spot, separately for each offense type $j \in \{l, h\}$. In the version presented here, we only consider interior solutions. The resource constraint implies that the adjudicator has to consider also the total number of cases that are adjudicated through either procedure. For a given p_{Tj} and p_{Cj} , $F_j(p_{Cj}) - F_j(p_{Tj})$ is the fraction of cases that are resolved at trial, and $1 - F_j(p_{Cj})$ is the fraction of cases that are convicted without trial.

The objective function of the adjudicator is to minimize the total expected cost of errors⁷

$$\min_{p_{Tj}, p_{Cj}} \sum_{j=l, h} \left[\int_0^{p_{Tj}} w_{aj} p_j f_j(p_j) dp + \int_{p_{Cj}}^1 w_{cj} (1 - p_j) f_j(p_j) dp \right],$$

subject to the resource constraint

$$R - \sum_{j=l, h} \{ [F_j(p_{Cj}) - F_j(p_{Tj})] c_T + [1 - F_j(p_{Cj})] c_C \} = 0.$$

The first integral expresses the total cost of wrongful acquittals - the cost of acquittal, weighted by the probability that the defendant with evidence p is actually guilty and the density of p among all defendants, for defendants with p below the threshold p_{Tj} . The second integral is the cost of wrongful convictions, integrated over ps above the threshold p_{Cj} . R denotes the available resources.

The first-order conditions are

$$\begin{aligned} p_{Tj} w_{aj} &= \lambda c_T \quad \text{and} \\ (1 - p_{Cj}) w_{cj} &= \lambda (c_T - c_C), \\ &\text{for } j = \{l, h\}. \end{aligned}$$

The first-order conditions are nearly identical to the optimality conditions (1) under the unconstrained problem except for the lambda. The Lagrange multiplier has a crucial interpretation here: by what factor are the marginal cost of wrongful conviction and acquittal greater than what they would have been if the adjudicator had the optimal (total cost minimizing) budget.

⁷This formulation is equivalent to the minimization of the sum of the cost of errors and the cost of procedure subject to the resource constraint (which would be a direct extension of the unconstrained problem), because the cost of procedure enter directly the resource constraint. However, this formulation is analytically simpler and the obtained values of λ have a more straightforward interpretation. It also allows studying situations when the adjudicator has an *excessive* budget.

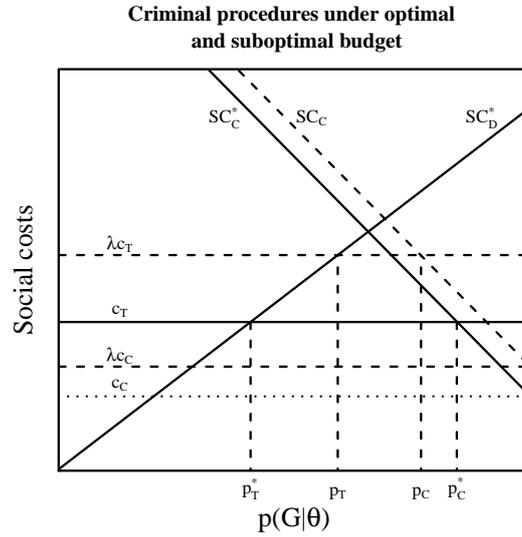


Figure 3:

It is optimal to tolerate a certain level of wrongful acquittals and conviction even in the unconstrained problem. Dropping a case implies an (expected) cost of wrongful acquittal $p w_a$. However, the nearest alternative—a trial—is also costly. It therefore pays to drop the cases with the very low inferred probability of guilt until the cost of wrongful acquittal for the marginal defendant ($p_j(G|p_{Tj})w_a$) are equal to the cost of trial c_T . Technically, $\lambda = 1$ in such a situation.

In the presence of a binding resource constraint, $\lambda > 1$. The adjudicator acts “as if” the cost of the trial were greater than they nominally are, because she simply cannot afford to have that many trials. As a consequence, the cost of wrongful acquittal of the marginal defendant, $p_j(G|p_{Tj})w_a$ are also greater. Too many defendants are dropped, and too many are convicted without trial, than would be socially optimal. This results in more erroneous verdicts than would be socially optimal. Interpreting λ from this perspective, it denotes the “marginal cost of justice”: it is equal to the marginal cost of wrongful convictions and acquittals due to reducing the criminal procedure budget by one dollar. The cost of the judicial process c_C and c_T effectively put a lower bound on the costs of errors that the society optimally chooses to tolerate. a reduction in the available resources pushes the costs of errors that are tolerated further up.

The first order-conditions also imply an important rule for the optimal use of the alternative procedure between the high-severity and low-severity offenses

$$p_h(G|p_{Th})w_{ah} = p_l(G|p_{Tl})w_{al} \quad \text{and} \quad (2)$$

$$[1 - p_h(G|p_{Ch})] w_{ch} = [1 - p_h(G|p_{Cl})] w_{cl}.$$

That is, the cost of error for the marginal defendants must be equalized across offense types.

4.2 Predictions

- The probability of a wrongful acquittal of the marginal defendant is lower for high-severity offenses than for low-severity offense, and likewise for the probability of wrongful conviction. This is implied directly by equation 2 and the fact that $w_{al} < w_{ah}$, $w_{cl} < w_{ch}$.
- A reduction in the available resources increases p_{Tj} (higher fraction of cases is dropped) and reduces p_{Cj} (higher fraction of cases is convicted without trial). It also increases the total social costs of errors. Figure 3 illustrates.
- A change in any parameter affects the optimal allocation across all types of offenses and all procedures through the resource constraint. Consider, for example, a reduction in the cost of conviction without trial. In the unconstrained problem, this would, of its own, reduce p_{Cj} for both offenses (more defendants convicted without trial). This channel of response is analogous to a substitution effect. However, because the cost of one procedure are decreased, more resources are effectively available. This generates an “income effect” whereby the adjudicator can afford to adjudicate more cases through the relatively more expensive but less erroneous trial. The income effect hence also induces a reduction in the fraction of cases that are dropped, and mitigates the increase in the fraction of cases that are convicted without trial. It reduces the overall sum of the cost of errors, even though more errors are committed in cases that were marginally shifted into conviction without trial.
- An increase in the cost of wrongful conviction for high-severity offenses w_{ch} causes an increase in the fraction of convictions without trial for low-severity offenses. The argument is analogous: An increase in w_{ch} produces a direct effect that pushes the adjudicator to reduce the cost of errors, hence shifting the high-severity cases from the conviction without trial to trial. That puts a strain on the resource constraint, and the adjudicator has to “give

in” on all remaining margins, including a greater reliance on the conviction without trial for the low-severity offenses.

- The threshold for conviction without trial for low-severity offenses, p_{Cl} is more elastic to changes in any parameter than p_{Ch} (under additional but very weak assumptions), that is for any parameter x

$$\frac{dp_{Cl}/dx}{dp_{Ch}/dx} > 1.$$

5 Implications for explaining the real-world criminal justice

5.1 Wider scope of plea bargaining.

Plea bargaining is used extensively in the United States to convict even the most serious offenders. In Europe, the alternative procedures are typically confined to low-severity. However, even within European countries that do have plea bargaining, more severe cases can be resolved via plea bargaining than via penal order or accelerated proceedings. Our model provides a useful framework for explaining this difference. If plea bargaining is indeed successful in revealing information about true guilt, then the combined signal (evidence plus confession) is highly informative and the conviction without trial can indeed be used even in very serious cases, as the model predicts. On the other hand, the verdicts under the European penal order and accelerated procedures are based largely on the information assembled by the police and prosecutors.⁸ The evidence is therefore less informative about true guilt. The model predicts that the use of the conviction without trial should be low particularly for high-severity offenses.

The American/European difference can also be explained by the cost of trials. In his comparison the American and German criminal procedure, Langbain (1979) argues that the American adversarial trial is much more complicated and time-consuming than the German inquisitorial trial. In the language of our model, c_T is higher in the United States than in Europe. It follows straightforwardly that then it is optimal to set low p_C (but also high p_T and adjudicate a smaller fraction of cases at trial.

⁸On the other hand, the prosecutors may assemble more evidence under the European alternative procedure because they have to present evidence that is sufficiently strong for a conviction under the penal order or accelerated procedure.

5.2 Spread of alternative procedures in Europe

Nevertheless, the alternative criminal procedures for low-severity offenses proliferated throughout Europe in the recent decades, and the model also offers a framework for explaining this phenomenon. The simplified procedure would be used more extensively (lower p_{CI}) if

- λ increases. That is, if the budgetary constraint on the criminal justice system became more severe, implementing a cheaper procedure is a rational response.
- The scope of the criminal law widens. Many countries expanded the criminalization of various administrative or regulatory violations in the last few decades. An increase in the number of cases to be adjudicated is analogous to a reduction in the available resources, holding the number of cases constant. Reduction of p_{CI} is an optimal reaction, particularly if the newly criminalized offenses are predominantly low-severity offenses.
- The police produces more informative arrests, particularly among low-severity offenses. The adjudicator then faces a more precise signal in the larger fraction of cases and can convict more without trial without incurring higher cost of wrongful conviction. The informativeness of arrests can improve through better forensic technology, e.g., better alcohol testing or street cameras. It can also increase (on average) through the composition of criminal cases, such as the increased criminalization of administrative violations discussed above. The identity of the offender of, for example, an environmental violation is typically immediately known, unlike the identity of the offenders of “classical” crimes such theft.
- A very intriguing explanation is offered by the prediction 3. The alternative procedures could have been adopted for low-severity crimes because the societies became *more* concerned with the cost of wrongful convictions. The growing respect for human rights and justice is captured in the model as increasing costs of wrongful convictions. But it is very plausible that the cost of wrongful conviction for severe crimes (e.g. murder) increased by more (particularly in absolute terms) than the cost of wrongful conviction for petty crimes such as pickpocketing. A greater fraction of convictions without trial for petty crimes is then an optimal response, despite the fact that it increases the cost of

wrongful conviction for such petty crimes. However, if avoiding wrongful convictions for very serious crimes is far more important, the adjudicator needs to shift resources from elsewhere in the system to assure that more of severe crimes are adjudicated at trial.

- The prediction 4 also implies greater sensitivity of p_{Tl} and p_{Cl} for low-severity offenses to any shock. The adjudicators and policy makers would hence be more willing to adjust the scope and rules of the simplified procedures for low-severity offenses as the need arises. The low-severity offenses naturally offer a greater scope for procedural experimentation while the serious offenses are prone to greater procedural stability.
- Finally, alternative procedures are more attractive if higher income decreases the relative cost of appeals to the defendants.

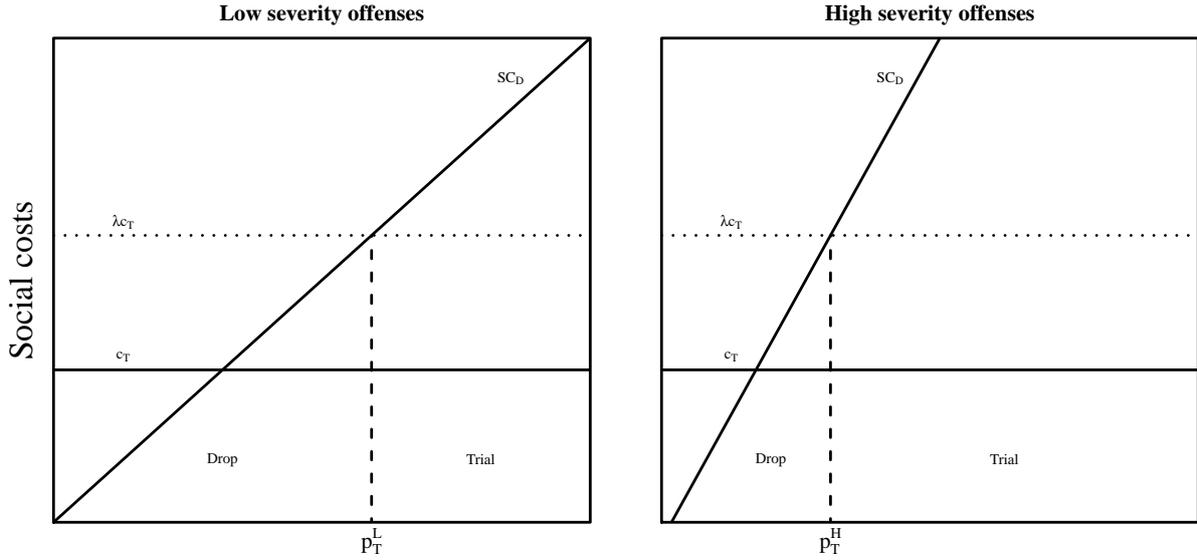
5.3 Consequences of introducing the alternative procedure

The standard justification for plea bargaining, or for the introduction of the alternative procedures in Europe, is that they release the enforcement resources from petty crimes hence allow the enforcers to concentrate on the truly complicated, serious crimes. Our model shows that such a beneficial effect, while present, is quantitatively limited. The optimal reallocation of resources leads primarily to an increase in the enforcement of petty crimes. This effect is robust to whether the alternative procedure is limited to petty crimes (as has been typical in most implementations in Europe) or extended to all crimes.

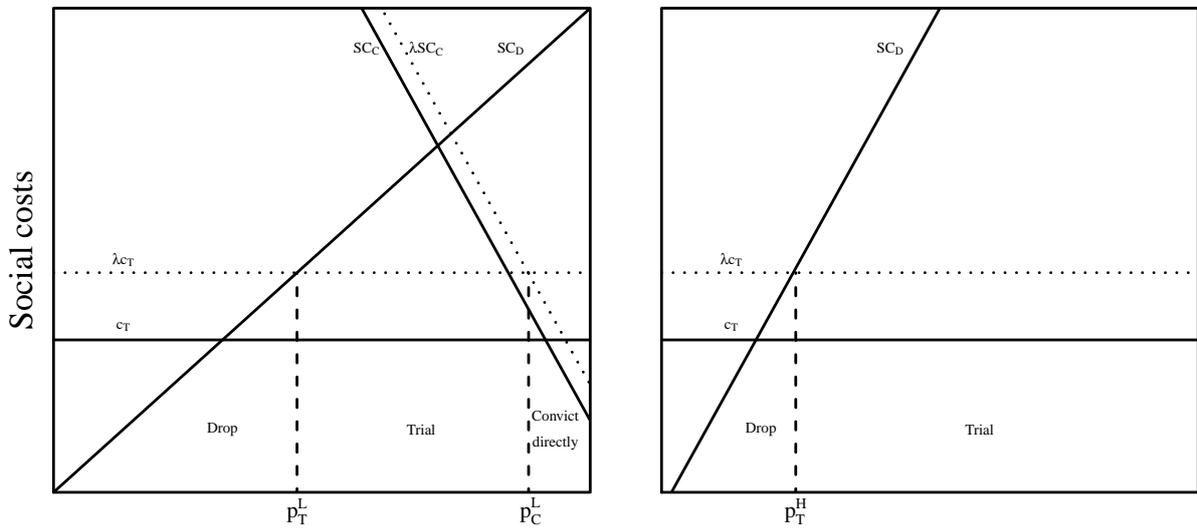
This argument is presented in Figure 4. While the resources invested into a cases are not explicitly modelled, the model well captures the allocation of resources in to serious/petty crimes by the fraction of serious/petty cases that are actually enforced either via trial or direct conviction. A reduction in p_{Tj} indicates an increase in enforcement of crime type j . In the initial situation in Figure 4 the alternative procedure is not available; cases can be either dropped or resolved at trial. It is optimal to allocate the cases with the highest p 's to the trial. The marginal value of additional resources is high (high λ_0) because the procedure is expensive and p_{Tl}^0 and p_{Th}^0 are high as well; a large fraction of cases of both types must be dropped.

When an alternative procedure is introduced for low-severity offenses only, the line SC_{Cl} , scaled up by the new λ_1 enters the trade-off for the low-severity offenses. Some low-severity

A: Simplified procedure not available



B: Simplified procedure available for low offenses only



C: Simplified procedure available for all offenses

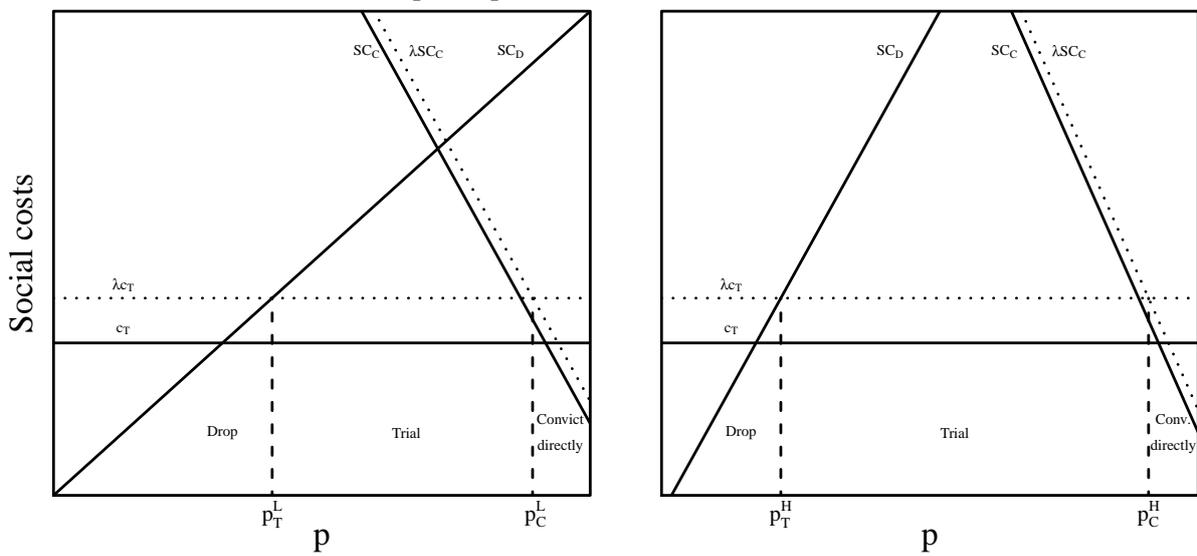


Figure 4: The effects of introducing direct conviction.

offenses can be directly convicted, which releases resources and reduces λ . As a result, p_{Tl}^0 and p_{Th}^0 fall, and fewer cases of each type are dropped. In that sense, the standard justification is correct, more resources are allocated to enforcing the high-severity offenses. However, the graph shows clearly that p_{Tl}^0 falls by more than p_{Th}^0 because the line SC_{Dh} depicting the social cost of wrongful acquittals for high-severity offenses, is steeper than the corresponding line SC_{Dl} while the reduction in λ is the same for both offense types.

Intuitively, the explanation is as follows: When resources are really scarce, the adjudicator has to make sure that they are allocated to cases where the cost of wrongful acquittal is the highest that is, the high-severity offenses with high p . Because the cost of errors rise steeply with p , a relatively large fraction of cases does go to trial but also that fraction is relatively inelastic with respect to the changes in the cost. When the alternative procedure becomes available, the main reason why the released resources are not allocated primarily to the high-severity offenses is that it was already optimal to allocate the existing resources to such cases in the first place. In the similar vein that increases in consumer's income are used to purchase goods with relatively low marginal utility, the released resources are disproportionately allocated to low-severity offenses.⁹

Extending the alternative procedure to high-severity offenses as well (a scenario marked by superscript 2 in Figure 4 is welfare-improving by construction - it expands the choice set. The strongest high-severity cases are now convicted directly as well, which releases resources and reduces λ further to λ_2 . As a result, p_{Tl}^0 and p_{Th}^0 fall further, fewer cases of each type are dropped, and again, the p_{Tl}^0 falls by more than p_{Th}^0 : relatively more low-severity cases are now enforced. The main difference from the previous scenario is that 1) overall more enforcement and 2) a smaller fraction of low-severity cases are directly convicted.

This implication of our model chills some of the alleged benefits of the alternative procedures. The main benefit is not a more vigorous enforcement of the serious crimes but rather more vigorous enforcement of the petty crimes. Those crimes were left unenforced in the absence of the alternative procedure, because there were not enough resources to enforce them and the cost of wrongful acquittals were still lower than the cost of wrongful acquittals for the serious crimes.

⁹Mathematically, this result follows directly from the prediction 5. The allocation thresholds are more sensitive to changes in any parameter for the low-severity offenses. The parameter change here is the reduction in the cost of the alternative procedure from infinity to c_c .

6 Extensions

6.1 Appealing the direct conviction

Simplified procedures are controversial as they carry lower standards of case examination compared to court trials. As a result, they increase the risk that an innocent person will be convicted. At the same time, simplified procedures usually allow the defendant to appeal, in which case a court reviews the case. How does the possibility of appeal affect the properties and the optimal use of simplified procedures? The main model can be easily extended to study these questions.

Assume that courts are again perfect and the defendant may appeal the decision. Assume also that social costs of appeal are c_T , that is the cost of trial, whereas the cost of appeal to the defendant are zero. As a result all (directly) convicted defendants who are innocent will appeal to the perfect court and there will be no wrongful convictions. The expected social costs of direct conviction are thus $pc_C + (1 - p)(c_C + c_T) = c_C + (1 - p)c_T$ and the adjudicator's optimization problem will be

$$\min_{p_{Tj}, p_{Cj}} \sum_{j=l,h} \int_0^{p_{Tj}} w_{aj} p_j f_j(p_j) dp,$$

subject to the resource constraint

$$R - \sum_{j=l,h} \left\{ [F_j(p_{Cj}) - F_j(p_{Tj})] c_T + [1 - F_j(p_{Cj})] c_C + \int_{p_{Cj}}^1 c_T (1 - p_j) f_j(p_j) dp \right\} = 0.$$

The first order conditions are then

$$p_{Tj} w_{aj} = \lambda c_T, \quad \text{and}$$

$$(1 - p_{Cj}) c_T = c_T - c_C,$$

implying

$$p_{Tj} = \frac{\lambda c_T}{w_{aj}}, \quad \text{and}$$

$$p_{Cj} = \frac{c_C}{c_T}.^{10}$$

The decision whether to push case before a trial or drop it, and thus the optimal standard of evidence for trial eligibility, are the same as in the case without appeal. However, the direct conviction no longer involves any trade-off between the adjudicator costs and potential wrongful conviction. As a result, the optimal standard of evidence for direct conviction is purely determined by the relative cost of this procedure compared to trial costs. Intuitively, knowing all innocent will appeal, it makes sense to directly convict if the savings, $c_T - c_C$, are greater than the cost of additional trials for those who appeal, $(1 - p)c_T$.

To summarize, in this setup: (i) introducing simplified procedure does not produce any wrongful convictions; (ii) budget shocks do not alter the optimality conditions for the use of the simplified procedure (assuming the budget allows for interior solution); (iii) budget shocks only affect the evidence standard required for trials. The real-world implication is that the possibility of appeal may eliminate most, if not all, costs related to wrongful conviction due to a less rigorous judicial process, provided the appellate court review is accessible at low cost to the defendant.

6.2 Imperfect courts

The assumption of perfect courts is – intentionally – a strong one. We also derived a model with imperfect courts in order to check the robustness of the model to this assumption. Introducing the court imperfection does not qualitatively alter the key results of the paper; it yields, however, a few additional subtle implications. For that reason, we only summarize the key intuitive points here and report the formal derivation in the appendix.

A court trial reveals more information about the case, and hence generate a more precise information about the defendant's guilt. Technically, more precise information means that $E[p'|p, G] > p$ and $E[p'|p, I] < p$, where p' denotes the inferred posterior probability (after trial) that the defendant is guilty. That is, the factually guilty defendants should, on average, leave the trial with a higher inferred probability of guilt than the initial p , and the innocent with a lower inferred probability. The link between p' and p may in principle be complicated function, dependant on the initial p and, crucially, on the ability of the court to reveal new information.

In order to keep the model simple and to capture the ability of the court, we assume that the revelation of new information at trial can have only two outcomes:

- The court reveals the full truth with probability ρ . In that case, the court convicts the factually guilty and releases the factually innocent, as in the perfect court case. No cost of wrongful conviction or acquittal is incurred.
- The court learns nothing new with probability $1 - \rho$. In that case, the court has to adjudicate the case – convict or acquit the defendant – based on the initial p . Like in the standard Andreoni (1991) framework, there is a critical level p_N such that the court decides to convict if $p > p_N$. A positive cost of either wrongful conviction or wrongful acquittal is incurred.

The parameter ρ simply captures the court's value added in revealing new information, and it also denotes the probability that an error will be incurred, should the case proceed to trial. The initial assumption of the perfect court is merely a special case with $\rho = 1$.

The results, however, are qualitatively the same as with the perfect courts. Quantitatively, the trade-off between trial and either dropping the case or convicting without trial is affected by the fact that the courts deliver an error-free verdict only with a probability. The relative benefit of trial is reduced, which leads to a greater use of conviction without trial in the optimum (and also fewer cases being dropped, because additional resources are saved by a greater use of conviction without trial). Intuitively, there is little reason to conduct a costly trial if it does not reveal much new information (i.e., its ρ is low).

The model with imperfect courts thus highlights the fundamental rationale for using court trials in the first place: they must generate new, more precise information. Second, it yields a testable prediction that trials would be used less and the alternative procedures more in legal systems where court trial perform a poor function in revealing new information.

7 Conclusions

We provided a general model that captures the key trade-offs between the adjudication of cases via court trials or alternative procedures that are less costly but also less precise. The model was

intentionally institution-free in order to capture the essential economics of the various procedural alternatives. We tend to think that these economic essentials are the first-order factors driving the use of the alternative procedures. We showed that the model provides a useful framework for thinking about broad differences in the criminal justice systems over time and across countries. Also, it provides a “workhorse” for incorporating additional features. For example, building the agency problems and institutional details of particular procedures would provide a useful comparative analysis of how, and under what conditions, the alternative procedures succeed or fail to achieve the optimal criminal justice outcomes.

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A Appendix

A.1 Imperfect courts

Preliminaries:

- Courts ascertain the truth with probability ρ .
- With prob. $1 - \rho$ they learn nothing, i.e. decide according to p .
- Let $p_H = w_c / (w_a + w_c)$, which is the probability beyond which courts should convict.¹¹

Possible outcomes: and costs

¹¹This comes from a situation when $p w_a = (1 - p) w_c$. Then solving for p gives $p = w_c / (w_a + w_c)$.

- drop case: w_a
- go to trial: $c_T + (1 - \rho)pw_a$, if $p < p_H$
- go to trial: $c_T + (1 - \rho)(1 - p)w_c$, if $p > p_H$
- convict w/o trial: $c_C + (1 - p)w_a$

$$\min_{p_{Tj}, p_{Cj}} \sum_{j=l,h} \left\{ \int_0^{p_{Tj}} w_{aj} p_j f_j(p_j) dp \right. \\ \left. + (1 - \rho) \left[\int_{p_{Tj}}^{p_{Hj}} w_{aj} p_j f_j(p_j) dp + \int_{p_{Hj}}^{p_{Cj}} w_{cj} (1 - p_j) f_j(p_j) dp \right] \right. \\ \left. + \int_{p_{Cj}}^1 w_{cj} (1 - p_j) f_j(p_j) dp \right\},$$

subject to the resource constraint

$$R - \sum_{j=l,h} \left\{ [F_j(p_{Cj}) - F_j(p_{Tj})] c_T + [1 - F_j(p_{Cj})] c_C \right\} = 0.$$

FOCs:

$$w_{aj} p_{Tj} f_j(p_{Tj}) - (1 - \rho) w_{aj} p_{Tj} f_j(p_{Tj}) - \lambda f_j(p_{Tj}) c_T = 0$$

$$w_{aj} p_{Tj} - (1 - \rho) w_{aj} p_{Tj} = \lambda c_T$$

$$\rho p_{Tj} w_{aj} = \lambda c_T$$

$$(1 - \rho) w_{cj} (1 - p_{Cj}) f_j(p_{Cj}) - w_{cj} (1 - p_{Cj}) f_j(p_{Cj}) - \lambda f_j(p_{Cj}) (c_C - c_T) = 0$$

$$(1 - \rho) w_{cj} (1 - p_{Cj}) - w_{cj} (1 - p_{Cj}) = \lambda (c_C - c_T)$$

$$-\rho w_{cj} (1 - p_{Cj}) = \lambda (c_C - c_T)$$

$$\rho (1 - p_{Cj}) w_{cj} = \lambda (c_T - c_C)$$

Solving for p_C

$$\rho (1 - p_{Cj}) w_{cj} = \lambda (c_T - c_C)$$

$$p_{Cj} = \frac{\lambda (c_C - c_T) + \rho w_{cj}}{\rho w_{cj}}$$

A.2 Appealing the less-than-trial conviction with imperfect court

Preliminaries: Zero cost of appeal to the def. → all innocent appeal.

- Courts ascertain the truth with probability ρ .
- With prob. $1 - \rho$ they learn nothing, i.e. decide according to p .
- Let $p_H = w_c/(w_a + w_c)$, which is the probability beyond which courts should convict.¹²

Possible outcomes: and costs

- drop case: w_a , for $p \in [0, p_T]$
- go to trial: $c_T + (1 - \rho)pw_a$, for $p \in [p_T, p_H]$
- go to trial: $c_T + (1 - \rho)(1 - p)[c_T + (1 - \rho)w_c]$, for $p \in [p_H, p_C]$ which can be written as $c_T[1 + (1 - \rho)(1 - p)] + (1 - \rho)^2(1 - p)w_c$,
- convict w/o trial: $c_C + (1 - p)[c_T + (1 - \rho)w_c] = c_C + (1 - p)c_T + (1 - \rho)(1 - p)w_c$, for $p \in [p_C, 1]$

$$\begin{aligned} \min_{p_{Tj}, p_{Cj}} \sum_{j=l, h} \left\{ \int_0^{p_{Tj}} w_{aj} p_j f_j(p_j) dp \right. \\ \left. + (1 - \rho) \left[\int_{p_{Tj}}^{p_{Hj}} w_{aj} p_j f_j(p_j) dp \right. \right. \\ \left. \left. + (1 - \rho) \int_{p_{Hj}}^{p_{Cj}} w_{cj} (1 - p_j) f_j(p_j) dp \right. \right. \\ \left. \left. + \int_{p_{Cj}}^1 w_{cj} (1 - p_j) f_j(p_j) dp \right] \right\}, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \min_{p_{Tj}, p_{Cj}} \sum_{j=l, h} \left\{ \int_0^{p_{Tj}} w_{aj} p_j f_j(p_j) dp \right. \\ \left. + (1 - \rho) \left[\int_{p_{Tj}}^{p_{Hj}} w_{aj} p_j f_j(p_j) dp + \int_{p_{Cj}}^1 w_{cj} (1 - p_j) f_j(p_j) dp \right] \right\} \end{aligned}$$

¹²This comes from a situation when $pw_a = (1 - p)w_c$. Then solving for p gives $p = w_c/(w_a + w_c)$.

$$+ (1 - \rho)^2 \int_{p_{Hj}}^{p_{Cj}} w_{cj}(1 - p_j)f_j(p_j)dp \},$$

subject to the resource constraint

$$R - \sum_{j=l,h} \left\{ [F_j(p_{Hj}) - F_j(p_{Tj})]c_T \right. \\ \left. + [F_j(p_{Cj}) - F_j(p_{Hj})]c_T + \int_{p_{Hj}}^{p_{Cj}} c_T(1 - \rho)(1 - p_j)f_j(p_j)dp \right. \\ \left. + [1 - F_j(p_{Cj})]c_C + \int_{p_{Cj}}^1 c_T(1 - p_j)f_j(p_j)dp \right\} = 0.$$

$$R - \sum_{j=l,h} \left\{ [F_j(p_{Cj}) - F_j(p_{Tj})]c_T + [1 - F_j(p_{Cj})]c_C \right. \\ \left. + \int_{p_{Hj}}^1 c_T(1 - p_j)f_j(p_j)dp - \int_{p_{Hj}}^{p_{Cj}} c_T\rho(1 - p_j)f_j(p_j)dp \right\} = 0.$$

FOCs:

$$w_{aj}p_{Tj}f_j(p_{Tj}) - (1 - \rho)w_{aj}p_{Tj}f_j(p_{Tj}) - \lambda f_j(p_{Tj})c_T = 0$$

$$w_{aj}p_{Tj} - (1 - \rho)w_{aj}p_{Tj} = \lambda c_T$$

$$\rho w_{aj}p_{Tj} = \lambda c_T$$

$$-(1 - \rho)w_{cj}(1 - p_{Cj})f_j(p_{Cj}) + (1 - \rho)^2 w_{cj}(1 - p_{Cj})f_j(p_{Cj}) \\ - \lambda [f_j(p_{Cj})(c_C - c_T) + c_T\rho(1 - p_{Cj})f_j(p_{Cj})] = 0$$

$$-(1 - \rho)w_{cj}(1 - p_{Cj}) + (1 - \rho)^2 w_{cj}(1 - p_{Cj}) = \lambda [(c_C - c_T) + c_T\rho(1 - p_{Cj})]$$

$$-(1 - \rho)\rho w_{cj}(1 - p_{Cj}) = \lambda [(c_C - c_T) + c_T\rho(1 - p_{Cj})]$$

$$\rho(1 - \rho)(1 - p_{Cj})w_{cj} = \lambda [c_T - c_C - \rho(1 - p_{Cj})c_T]$$

Solving for p_{Cj}

$$\rho(1 - \rho)w_{cj} - \rho(1 - \rho)p_{Cj}w_{cj} = \lambda[c_T - c_C - \rho c_T + \rho p_{Cj}c_T]$$

$$\rho(1 - \rho)w_{cj} - \rho(1 - \rho)p_{Cj}w_{cj} - \lambda \rho p_{Cj}c_T = \lambda[c_T - c_C - \rho c_T]$$

$$p_{Cj}[-\rho(1 - \rho)w_{cj} - \lambda c_T] = \lambda[c_T - c_C - \rho c_T] - \rho(1 - \rho)w_{cj}$$

$$p_{Cj} = \frac{\rho(1 - \rho)w_{cj} - \lambda[c_T - c_C - \rho c_T]}{\rho(1 - \rho)w_{cj} + \lambda c_T}$$