Stochastic Cooperation of Non-Mutually Dependent Sellers*

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Abstract

We consider the strategic interaction between two sellers facing demand of different customers who are related by a one-sided externality: only the price of one seller affects the other seller's demand. Because of this externality, both sellers can gain by maximizing their joint profit and sharing it. We assume that cooperation via joint profits maximization is possible only with an exogenously given probability which, however, is varied systematically since we are interested whether sellers would want to render it more likely, if this is possible. So we derive how prices and expected profits of the sellers depend on the likelihood of cooperation as well as on the systematically varied externality parameter. Whereas the expected profit of the monopolist increases monotonically with the cooperation probability, the expected profits of the dependent seller is surprisingly U-shaped: decreasing with more likely cooperation when its probability is small and gaining with more likely cooperation when probability is large. So both sellers are ex-ante better off only when the cooperation probability is high enough. If sellers can only slowly move from independent pricing behavior to cooperation, for instance, to not trigger the attention of the antitrust authority, they may not be able to get off the ground what would justify that antitrust monitoring of one-sided dependent sellers is largely neglected.

1 Introduction

On markets with interdependent sellers, their obvious price externalities allow for gains from cooperation, for instance, in form of profitable cartelization. The literature has thoroughly investigated such collusion, cartel stability, and the effectivity of antitrust interventions. In models of horizontally

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and vertically differentiated products, the analyses have highlighted the role of relevant features of the economic environment, for instance, the degree of product differentiation, cost asymmetries and demand volatility (for surveys see Feuerstein, 2005, and Marini, 2018). While initial studies have focused on horizontal and vertical product differentiation separately, more recent papers have extended the analysis to settings with both horizontal and vertical product differentiation, see for instance Symeonidis (1999) and Sen et al. (2024).

We depart from these analyses of mutually interdependent sellers since we consider a one-sided dependency of sellers whom we assume to collude with an exogenously given probability.[1] In the dyadic set with one independent and on dependent seller, the pricing of the independent seller affects the demand, hence also the profits of the dependent seller, whereas the reverse does not hold. As an example of a one-sided positive externality, consider the case of AI chatbots which supply services (e.g., tailored ready-made codes with just few instructions) that increase the demand for scientific or professional software packages. As an example of a one-sided negative externality, consider the case of fashion goods like luxury or branded products, whose demand is, at least to some extent, by the pricing of unbranded ones whereas the demand for the lower quality, unbranded products is likely inversely affected by the price of the high quality ones. In these settings, collusion, while profitable, may be hampered by the complexity of the communication and coordination tasks needed to sustain it.[2] Therefore, successful cartelization may be regarded as an uncertain outcome of the strategic interaction among interdependent sellers, who will receive non-cooperative profits in case collusion fails.

Specifically, we consider one-shot interactions of sellers which collude via competitive and collusive prices, as in Nash (1953), i.e., in form of collusive pricing letting them share evenly the surplus from cartelization, given by the difference between the maximal joint profit and the sum of non-cooperation profits, but only with an exogenously given probability. What our analysis of such a dyadic interaction shows is that the expected profits of the monopolist increases monotonically with the cooperation probability, whereas the expected profits of the dependent seller evolve in a U-shaped way, i.e., are lower (higher) for low (large) probabilities: both sellers are thus only better off when the exogenous probability of sharing maximal joint profits is sufficiently large.[3]

Bargaining with variable threat is a familiar topic in bargaining theory ever since Nash (1953)

[1] Standard duopoly models with product differentiation mainly follow the seminal approach by Dixit (1979) and consider two-sided dependency, i.e., markets in which both goods have a non-zero degree of substitutability.

[2] When it occurs at the expenses of consumers, cartelization may also be prevented by (possibly stochastic) interventions of antitrust authorities.

[3] a me sembra che Werner qui dia per scontato che il conflict point per $\beta = 0$ sia uguale a quello per $\beta = 1$ e che pertanto i profitti di equilibrio per $\beta = 1$ sono più alti per entrambi rispetto a quelli per $\beta = 0$. Cosa che invece non è scontata.

and its application to duopoly markets (Mayberry et al, sempre da verificare questa citazione). Although it seems more natural and intuitive to let parties determine their disagreeemnt threats after failing to achieve an agreement, Nash assumes commitment to threats before bargaining. It is therefore not surprising that one has explored what happens instead when there is no such precommitment power (e.g., Kaneko and Mao, 1996).

We do not confine ourselves to comparing bargaining with and without precommiting to threats. Instead—like Ferstlman and Seidmann (1991), who render delayed agreements more costly, and Muthoo (1992), who assumes preannounced threats to be possibly revocable, we focus also on the generic intermediate cases when reaching an agreement and ending up in conflict where both occur with positive probability. Actually we are mainly interested in whether one-sidedly related sellers would be gradually increasing their cooperation probability since, with antitrust monitoring drastically switching from competition of sellers to cooperation is likely noticed and prevented.

According to the literature and field evidence antitrust authorities seem to exclusively monitor markets with mutually dependent sellers. Our market setting instead features two non-mutually dependent sellers, one monopolist whose price affects the demand level of only one dependent seller. Also in case of such one-sided dependency there can be large cooperation incentives whose, especially in case of drastically switching from independent to cooperative pricing assumes exogenously given, but continuously varying market cooperation probability. We capture cooperation as both sellers equally sharing their cooperation surplus, i.e., the difference of joint profits cooperation minus the sum of both sellers' disagreement profits. Due to possible side payments cooperation can rely on joint profit maximization by analyzing the whole hybrid game class of random cooperation probabilities we can show an, in our view, quite surprising result: only the monopolist, but not the dependent seller, is always gaining when the cooperation probability gradually increases. Actually, this might be one reason why antitrust monitoring widely neglects market situations with non-mutually dependent sellers: the dependent seller would experience losses from more likely cooperating and terminate cooperation or, at least, prevent for their increases and thus more harms for customers.

The paper is organized as follows: Section 2 describes the stochastic cartelization setup. Section 3 describes and comments the best responses and equilibrium prices. In Section 4 we derive the equilibrium profits and how they are affected by the probability of cartelization. Section 5 concludes.

2 Stochastic cartelization

One independent seller, M, and one dependent seller, D, face the following (linear) demand functions:

$$x_M(p_M) = \gamma - p_M \quad \text{and} \quad x_D(p_M, p_D) = (\gamma + \alpha p_M) - p_D,$$

in which, for $i = M, D$, x_i and p_i denote the demand for and the price of seller i, respectively.[4] When $\alpha = 0$, both sellers are independent monopolists, whereas when $\alpha \neq 0$ there are price spillover effects: the demand for D increases or decreases with p_M when $\alpha > 0$ or $\alpha < 0$, respectively.

If sellers behave non-cooperatively, they independently and simultaneously choose (p_M, p_D) to maximize their own market profits, which, since we normalize production costs to zero, are equal to:

$$\pi_M(p_M) = p_M x_M(p_M) \quad \text{and} \quad \pi_D(p_M, p_D) = p_D x_D(p_M, p_D). \tag{1}$$

Instead of behaving non-cooperatively, the sellers may form a cartel and agree to maximize the joint profits and share equally the surplus. In this case, we do not rely on exogenous conflict payoffs, but we assume that sellers non-cooperatively choose prices to be irrevocably used in case cartelization fails. In the terminology of bargaining with variable threats (see Nash, 1953),[5] the non-cooperation prices determine the conflict profits which determines the surplus to be shared in case cartelization occurs.

The maximal joint profits, denoted by Σ^J, are attained by choosing prices (p_M^J, p_D^J) equal to:[6]

$$p_M^J = p_D^J = \gamma/(2 - \alpha), \tag{2}$$

which lead to:[7]

$$\Sigma^J = \pi_M(p_M^J) + \pi_D(p_M^J, p_D^J) = \gamma^2/(2 - \alpha). \tag{3}$$

The surplus consists of the difference between the maximal joint profits and the conflict profits, $S(p_M, p_D) = \Sigma^J - \pi_M(p_M) - \pi_D(p_M, p_D)$.

If cartelization occurs, the sellers agree to a payoff equal to the own conflict profits plus half of

[4] To guarantee non-negative demand, p_M is restricted to the interval $[0, \gamma]$ and p_D to the interval $[0, \gamma + \alpha p_M]$. Furthermore, α is restricted to the interval $[-1, 1]$ by assumption.

[5] The original Nash (1953) model with variable threats (see also van Damme, 1991, Ch. 7.8) has been extended to more than two players by Kaneko and Mao (1996), while Bolt and Houba (1998) have considered an explicit dynamic bargaining stage.

[6] See Appendix A.1

[7] See Appendix A.2

the surplus. Therefore, in this case the profits of M and of D are equal to, respectively:

$$\pi_M(p_M) + \left(\frac{1}{2}\right) S(p_M, p_D) \quad \text{and} \quad \pi_D(p_M, p_D) + \left(\frac{1}{2}\right) S(p_M, p_D) \tag{4}$$

As explained in the Introduction, we consider a hybrid model in which sellers cooperate with an exogenously given probability β, with $0 \leq \beta \leq 1$. In this context of stochastic cartelization, the expected profits of the sellers depend on the possibility that cartelization occurs or not. If cartelization fails, sellers receive their non-cooperative profits (see (1)), while they attain the cooperative profits (see (4)) if cartelization succeeds. Taking the success or failure of cartelization into account, the expected profits of M are equal to:

$$\begin{aligned}
\hat{\pi}_M(p_M, p_D) &= \beta \overbrace{\left[\pi_M(p_M) + \left(\frac{1}{2}\right) S(p_M, p_D)\right]}^{\text{success}} + (1-\beta) \overbrace{\pi_M(p_M)}^{\text{failure}} \\
&= \pi_M(p_M) + \left(\frac{\beta}{2}\right) S(p_M, p_D) \\
&= \left(\frac{\beta}{2}\right) \Sigma^J + \left(1 - \frac{\beta}{2}\right) \pi_M(p_M) - \left(\frac{\beta}{2}\right) \pi_D(p_M, p_D).
\end{aligned} \tag{5}$$

while those of D are given by:

$$\begin{aligned}
\hat{\pi}_D(p_M, p_D) &= \beta \overbrace{\left[\pi_D(p_M, p_D) + \left(\frac{1}{2}\right) S(p_M, p_D)\right]}^{\text{success}} + (1-\beta) \overbrace{\pi_D(p_M, p_D)}^{\text{failure}} \\
&= \pi_D(p_M, p_D) + \left(\frac{\beta}{2}\right) S(p_M, p_D) \\
&= \left(\frac{\beta}{2}\right) \Sigma^J + \left(1 - \frac{\beta}{2}\right) \pi_D(p_M, p_D) - \left(\frac{\beta}{2}\right) \pi_M(p_M).
\end{aligned} \tag{6}$$

The second line in (5) and (6) shows that the expected profits can be expressed as the sum of the conflict profits, which accrues to sellers with certainty, and of the expected value of the surplus, whereas the third line shows that the expected profits depend on (p_M, p_D) via a weighted difference between own and other's non-cooperative profits, with weights equal to $1-\beta/2$ and $\beta/2$, respectively, which depend on the probability of the success of cartelization.

3 Best responses and equilibrium prices

Given β and the other price, the sellers simultaneously chooses the own price to maximize the expected profits. The best response $\hat{p}_M(p_D)$ of seller M and $\hat{p}_D(p_M)$ of seller D statisfy the following conditions, respectively:[8]

$$\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_M(\alpha,\beta)}{\partial p_M}-\left(\frac{\beta}{2}\right)\frac{\partial \pi_D(\alpha,\beta)}{\partial p_M}=0, \tag{7}$$

$$\frac{\partial \pi_D(\alpha,\beta)}{\partial p_D}=0, \tag{8}$$

and therefore are given by:

$$\hat{p}_M(p_D)=\frac{\gamma}{2}-\frac{\alpha\beta p_D}{2(2-\beta)} \quad \text{and} \quad \hat{p}_D(p_M)=\frac{\gamma}{2}+\frac{\alpha p_M}{2}. \tag{9}$$

When $\beta=0$, i.e., when cartelization fails, the best response of seller M does not depend on p_D because of the hypothesis of one-sided dependency. In this case, the profit maximizing price of M is equal to $\gamma/2$ and the associated non-cooperative profits are $\gamma^2/4$. When $\beta>0$, i.e., when cartelization is successful, the best response of seller M depends on β via the sign of α: for given p_D, when α is positive (negative) seller M reduces (increases) its price relative to $\gamma/2$, and this effect is stronger when the likelyhood of successful cartelization is higher. This follows since seller M aims to maximize $(1-\beta/2)\pi_M(p_M)-(\beta/2)\pi_D(p_M,p_D)$, given p_D. To this end, (s)he chooses p_M to reduces the demand for, hence the profits of, seller D by exploiting the price spillover effects which depend on the sign of α. However, when choosing $p_M\neq\gamma/2$, also the own profits of seller M decrease (relative to $\gamma^2/4$). The best response $\hat{p}_M(p_D)$ optimally trades off the reduction in $\pi_M(p_M)$ for the reduction in $\pi_D(p_M,p_D)$, taking into account the likelyhood of cartelization via the weights in (5).

The best response $\hat{p}_D(p_M)$ of seller D, on the other hand, does not depend on β. This follows from the hypothesis of one-sided dependency: seller D chooses p_D to maximize $(1-\beta/2)\pi_D(p_M,p_D)-(\beta/2)\pi_M(p_M)$, given p_M; however, since π_M does not depend on p_D, this is the same as maximizing $\pi_D(p_M,p_D)$.

Given (9), the equilibrium prices are equal to:[9]

$$p_M(\alpha,\beta)=\frac{\gamma\left[4-(2+\alpha)\beta\right]}{8-(4-\alpha^2)\beta} \quad \text{and} \quad p_D(\alpha,\beta)=\frac{\gamma(2+\alpha)(2-\beta)}{8-(4-\alpha^2)\beta}. \tag{10}$$

[8] See Appendix B.1

[9] See Appendix B.2

It is interesting to observe that one has:[10]

$$p_M(\alpha,\beta) - p_D(\alpha,\beta) = -\frac{2\alpha}{8-(4-\alpha^2)\beta}. \tag{11}$$

Therefore, $p_M(\alpha,\beta) > p_D(\alpha,\beta)$ when $\alpha < 0$ and the opposite occurs when $\alpha > 0$: consistently with the effects of the externality, the equilibrium price of seller M is higher (lower) than the price of seller D when there are negative (positive) price spillovers. Furthermore, for $i = M, D$ one has:

$$p_i(\alpha,\beta) = p_i^J \quad \text{if } \alpha = 0, \quad \text{and} \quad p_i(\alpha,\beta) \neq p_i^J \quad \text{otherwise.} \tag{12}$$

This follows since, when there are no price spillover effects, the maximization of expected profits is equivalent to the maximization of the seller's own profit regardless of the probability of successful cartelization and, in turn, this is equivalent to the maximization of the aggregate profits of the sellers.

Regarding how the equilibrium prices change with the probability of successful cartelization, from (10) it follows that:[11]

$$\frac{\partial p_M(\alpha,\beta)}{\partial\beta} = \frac{-4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2} \tag{13}$$

$$\frac{\partial p_D(\alpha,\beta)}{\partial\beta} = \frac{-2\alpha^2(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\gamma < 0 \tag{14}$$

Hence, $\partial p_M(\alpha,\beta)/\partial\beta$ depends on α, it is negative when $\alpha > 0$ and positive when $\alpha < 0$. To get an intuition for this result, observe that, when β increase, the weight on the negative component of the expected profits, i.e., $\pi_D(p_M, p_D)$, increases (see (5)), while the weight on the positive component, i.e., $\pi_M(p_M)$, decreases. Therefore, seller M adjusts p_M to reduce $\pi_D(p_M, p_D)$ since this has a greater positive effect on its expected profits than the associated negative effect due to the reduction in $\pi_M(p_M)$, and the sign of the externality determines whether this implies an increase or a decrease in p_M. Regarding $\partial pd(\alpha,\beta)/\partial\beta$, it is negative regardless the sign of α. The intuition for this results follows from the fact that, as β increases, seller M adjust p_M to depress the demand of seller D, who in turn reduces its price p_D.

[10]See Appendix B.3

[11]See Appendix B.4

4 Equilibrium profits

For $i = M, D$, let $\pi_i(\alpha, \beta)$ denote the equilibrium conflict profits, i.e., the conflict profits evaluated at the equilibrium prices, so that $\pi_M(\alpha, \beta) = \pi_M(p_M(\alpha, \beta))$ and $\pi_D(\alpha, \beta) = \pi_D(p_M(\alpha, \beta), p_D(\alpha, \beta))$. Furthermore, let $S(\alpha, \beta)$ denotes the surplus evaluated at the equilibrium prices, i.e., $S(\alpha, \beta) = \Sigma^J - \pi_M(\alpha, \beta) - \pi_D(\alpha, \beta)$. Given the equilibrium prices (10), one has:[12]

$$S(\alpha, \beta) = \frac{(2+\alpha)(4\alpha^2\gamma^2)}{(2-\alpha)[8-(4-\alpha^2)\beta]^2}. \tag{15}$$

Therefore, $S(\alpha, \beta) > 0$ for every $\beta \in (0, 1)$ and all $\alpha \in (-1, 1)$, with $\alpha \neq 0$. The same conclusion follows from (12) which, when $\alpha \neq 0$, implies $\Sigma^J > \pi_M(\alpha, \beta) + \pi_D(\alpha, \beta)$ by a direct revealed preference argument.

For $i = M, D$, let $\hat{\pi}_i^*(\alpha, \beta)$ denote the expected profits of seller i evaluated at the equilibrium prices, i.e.,

$$\hat{\pi}_i^*(\alpha, \beta) = \pi_i(\alpha, \beta) + \left(\frac{\beta}{2}\right) S(\alpha, \beta) \tag{16}$$

The equilibrium profits of seller M unambiguosly increase with the probability of successful cartelization; this follows since one has:[13]

$$\frac{\partial \hat{\pi}_M^*(\alpha, \beta)}{\partial \beta} = \left(\frac{1}{2}\right) S(\alpha, \beta), \tag{17}$$

which is positive due to $S(\alpha, \beta) > 0$. This implies, in particular, that $\hat{\pi}_M^*(\alpha, 1) > \hat{\pi}_M^*(\alpha, 0)$, so that in the limit case of successful cartelization, i.e., $\beta = 1$, the profits of the (independent) seller M are larger than in the limit case of conflict, i.e., $\beta = 0$.

Differently from what happen to seller M, and rather surpisingly, the equilibrium expected profits of seller D do not monotically increase with the probability of successful cartelization. this follows since one has:[14]

$$\frac{\partial \hat{\pi}_D^*(\alpha, \beta)}{\partial \beta} = \left(\frac{1}{2}\right) S(\alpha, \beta) - (1-\beta)\frac{\partial S(\alpha, \beta)}{\partial \beta} \tag{18}$$

The sign of this derivative cannot be promptly ascertained, since the first term is positive and the

[12]See Appendix C.1

[13]See Appendix C.2

[14]See Appendix C.3

second is negative, due to:[15]

$$\frac{\partial S(\alpha,\beta)}{\partial\beta} > 0. \tag{19}$$

However, the following proposition shows that, for β small enough, the second term is larger than the first, so that the expected profits are descreasing, while the opposite occur for β large enough:

Proposition 1. *For given α, the equilibrium expected profits of seller D are decreasing for $\beta < \bar{\beta}$ and increasing for $\beta > \bar{\beta}$, where*

$$\bar{\beta} = \frac{4(2-\alpha^2)}{3(4-\alpha^2)} \in (0,1) \tag{20}$$

Proof. See Appendix C.5. ■

Furthermore, the increase when β is large enough more than compensate the loss when β is small, so that eventually also for seller D the profits in the limit case of successful cartelization, i.e., $\beta = 1$, are higher than in the limit case of conflict, i.e., $\beta = 0$:[16]

$$\hat{\pi}^*_D(\alpha,1) > \hat{\pi}^*_D(\alpha,0). \tag{21}$$

To get an intuition for the different reaction of expected profits to changes in β, from (16) one has:

$$\frac{\partial\hat{\pi}^*_i(\alpha,\beta)}{\partial\beta} = \underbrace{\left(\frac{1}{2}\right) S(\alpha,\beta)}_{\text{direct}} + \underbrace{\frac{\partial\pi_i(\alpha,\beta)}{\partial\beta} + \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta}}_{\text{indirect}}. \tag{22}$$

The total effect of the change of β is the sum of a direct effect when holding $p_M(\alpha,\beta)$ and $p_D(\alpha,\beta)$, hence the surplus, constant, and an indirect effect which accounts for how conflict profits and surplus change with β via the induced adjustment in $p_M(\alpha,\beta)$ and $p_D(\alpha,\beta)$. While the direct effect is equal across sellers, the indirect effect differs between them: it is null for M (see (17)) and negative for D (see (18)). To dig deeper into this difference, which accounts for the different reaction of expected profits to changes in the likelyhood of cartelization, use (5) and (6) rewrite the indirect effects only in terms of the conflict profits:

$$\frac{\partial\pi_M(\alpha,\beta)}{\partial\beta} + \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta} = \left(1-\frac{\beta}{2}\right)\frac{\partial\pi_M(\alpha,\beta)}{\partial\beta} - \left(\frac{\beta}{2}\right)\frac{\partial\pi_D(\alpha,\beta)}{\partial\beta} \tag{23}$$

$$\frac{\partial\pi_D(\alpha,\beta)}{\partial\beta} + \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta} = \left(1-\frac{\beta}{2}\right)\frac{\partial\pi_D(\alpha,\beta)}{\partial\beta} - \left(\frac{\beta}{2}\right)\frac{\partial\pi_M(\alpha,\beta)}{\partial\beta} \tag{24}$$

[15]See Appendix C.4

[16]See Appendix C.6

Conflict profits change because of the adjustment in equilibrium prices, and therefore one has:

$$\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_M(\alpha,\beta)}{\partial \beta}-\left(\frac{\beta}{2}\right)\frac{\partial \pi_D(\alpha,\beta)}{\partial \beta}=$$

$$\underbrace{\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_M(\alpha,\beta)}{\partial p_M}\frac{\partial p_M}{\partial \beta}-\left(\frac{\beta}{2}\right)\frac{\partial \pi_D(\alpha,\beta)}{\partial p_M}\frac{\partial p_M}{\partial \beta}}_{(7)\Rightarrow=0}-\left(\frac{\beta}{2}\right)\underbrace{\frac{\partial \pi_D(\alpha,\beta)}{\partial p_D}\frac{\partial p_D}{\partial \beta}}_{(8)\Rightarrow=0} \tag{25}$$

$$\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_D(\alpha,\beta)}{\partial \beta}-\left(\frac{\beta}{2}\right)\frac{\partial \pi_M(\alpha,\beta)}{\partial \beta}=$$

$$\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_D(\alpha,\beta)}{\partial p_M}\frac{\partial p_M}{\partial \beta}-\left(1-\frac{\beta}{2}\right)\underbrace{\frac{\partial \pi_D(\alpha,\beta)}{\partial p_D}\frac{\partial p_D}{\partial \beta}}_{(8)\Rightarrow=0}-\left(\frac{\beta}{2}\right)\frac{\partial \pi_M(\alpha,\beta)}{\partial p_M}\frac{\partial p_M}{\partial \beta} \tag{26}$$

Since changes are evaluated at the (interior) solution to the sellers' own profits maximization problem, the corresponding derivatives are equal to zero. Therefore, for seller M the (first order) effect of the change in equilibrium prices is null because of the combined role of one-sided dependency, which implies that the change in p_D has no direct spillover on π_M, and profit maximization, which implies that (first order) effects from own prices on profits of both sellers are null. For seller D the situation is different: while the (first order) effect of the change in p_D is null because of the profit maximization argument, the (first order) effect of the change in p_M is not nullified and, as shown in (18), is indeed equal to $-(1-\beta)\partial S(\alpha,\beta)/\partial\beta$.

In turn, this indirect effect is the weighted difference between the variation of own conflict profits and conflict profits of the other seller with weights that reflects the probability of cartelization, i.e., $(1-\beta/2)$ for own conflict profits and $(\beta/2)$ for the other one, respectively (see equations (23) and (24)). Both conflict profits decrease as β increases,[17] but the absolute value of the variation is higher for D than for M for all β but for $\beta=1$, when they are equal. Moreover, $(1-\beta/2)$ decreases with β while $(\beta/2)$ increases. For seller M, the weights offset the different magnitude in the change of the conflict profits, so that the overall indirect effect is null. For D, instead, the absolute value of variation of its conflict profits and the associated weight are higher than for M. Hence, the indirect effect is negative but for $\beta=1$, when both the variations of the conflict profits and the weights are pairwise equal.

In the following, we rely on Figures 1–3 to further discuss our results. To illustrate the relationship

[17] See Appendix C.7.

between the price of M and the expected profits of both sellers let us consider Figure 1 which focuses on $\gamma = 100$, $\alpha = 0.9$ (upper panel) and $\alpha = -0.9$ (lower panel).[18] On the x-axis we represent p_M while on the y-axis we report the profits in case cartelization fails and in case it succeeds, as well as the equilibrium expected profits. Indeed, given p_M, by considering the associated best response of D ($\hat{p}_D(p_M)$), we determine the conflict profits for M and D which are represented by the dashed curves in green and brown, $\pi_M(p_M)$ and $\pi_D(p_M, \hat{p}_D(p_M))$, respectively. The continuous violet line represents the sum of the non-cooperative profits, its maximum (Σ^J) is attained when $p_M = p_M^J$. The continuous green and brown curves represent the profits in case cartelization succeeds for M and D, $\pi_M(p_M) + S(p_M, \hat{p}_D(p_M))/2$ and $\pi_D(p_M, \hat{p}_D(p_M)) + S(p_M, \hat{p}_D(p_M))/2$, respectively. To determine the equilibrium expected profits, for given $\beta \in [0, 1]$, we consider the equilibrium prices and represent the expected profits with green and brown circles for M and D, respectively, $\hat{\pi}_M^*(\alpha, \beta)$ and $\hat{\pi}_D^*(\alpha, \beta)$. Since the expected profits are a convex combination of the conflict profits and the profits in case cartelization succeeds, for a given $p_M(\alpha = 100, \beta)$, they lie on the segments that connects them. Before presenting the two subfigures, let us recall that, when α is positive then M undercuts the price relative to $\gamma/2$ whereas for $\alpha < 0$ the equilibrium price of M, $p_M(\alpha, \beta)$, is larger prices than $\gamma/2$.

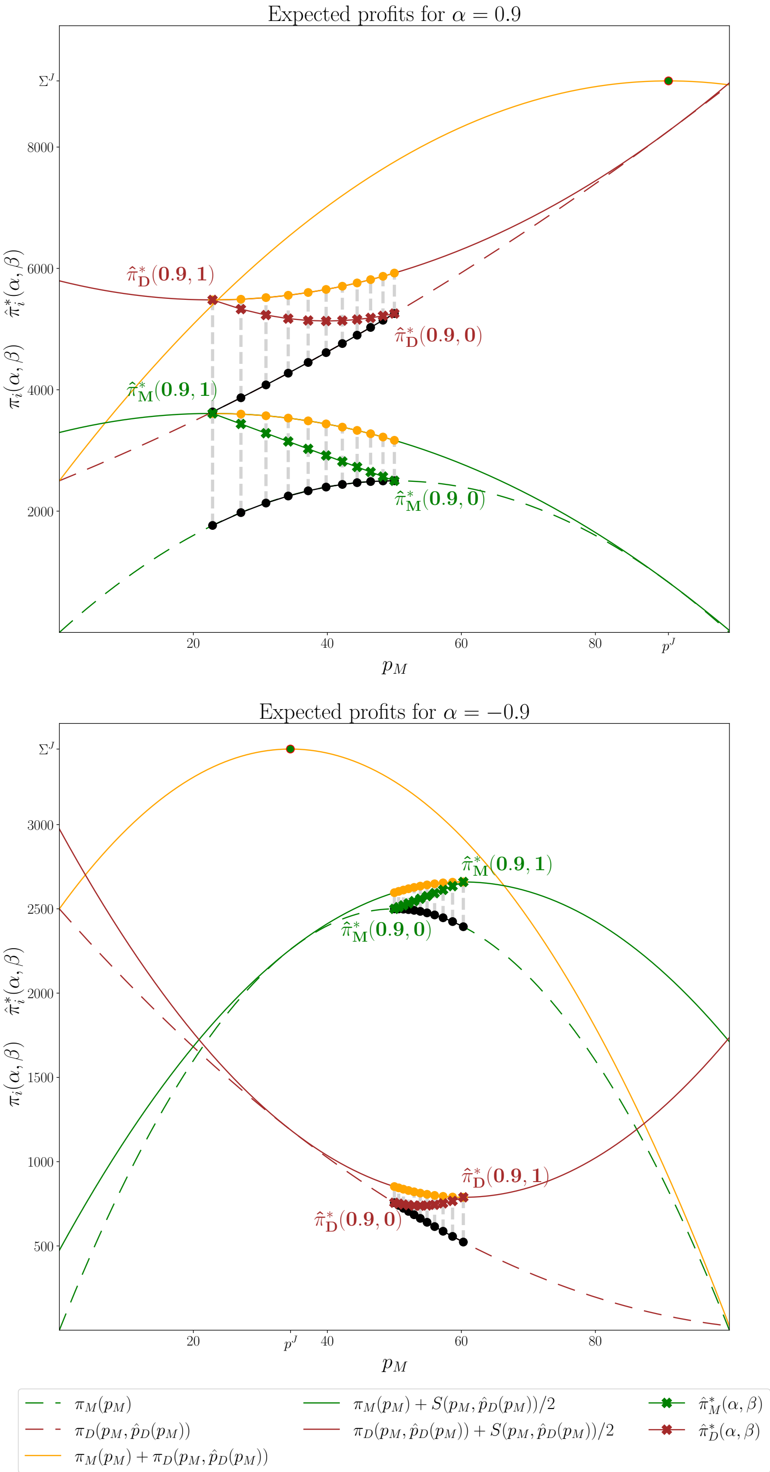
In the upper panel ($\alpha = 0.9$ and $\gamma = 100$), when $\beta = 0$ the equilibrium variables coincide with the equilibrium non-cooperative ones, i.e., the equilibrium prices are $p_M(\alpha = 100, \beta = 0) = 50 (= \gamma/2)$ and $p_D(\alpha, \beta) = 72.5$, the conflict profits are $\pi_M = 2500$ and $\pi_D = 5256.25$ which coincide with the expected profits since cartelization never succeeds. As argued above when β increases, the M-seller undercuts his price below the equilibrium non-cooperative one, by so doing both conflict profits $\pi_M(\alpha, \beta)$ and $\pi_D(\alpha, \beta)$ decrease but as it is apparent from the figure the conflict profits of D decreases more than the one of M and this allows M to seize a relevant part of the surplus. As β increase, $p_M(\alpha, \beta)$ decreases and the difference in the reduction of the conflicts profits shrinks. As a result, while the expected profits of M increase with β, the one of D at first decrease and then increase, reaching a maximum in $\beta = 1$.

In the lower panel ($\alpha = -0.9$ and $\gamma = 100$) the conflict profits of D are decreasing in p_M. Hence, by marginally increasing its price above the market one, M can induce a reduction in both conflict profits but triggers a higher decrease in the ones of D. As β increases, p_M increases and the difference in the variations of the two conflict payoffs is going to weaken. Therefore, also with a negative α, the expected profits of M increase with β while the one of D at first decreases and then increases, reaching a maximum in $\beta = 1$.

Figures 2–3 provide a representation of the profit-possibility frontier with and without side

[18]We consider $|\alpha| = 0.9$ to have noticeable effects on the expected profits of the sellers.

Figure 1: Equilibrium expected profits for $\gamma = 100$, $\alpha = -0.9$ and different values of β.



payments and of the equilibrium expected profits of the firm as β varies. We report on the x-axis and y-axis the expected profits of M and D, respectively. Curve $\mathcal{F}$ represents the profit-possibility frontier while $\mathcal{F}^{SP}$ the profit-possibility frontier with side payments. For given β, Γ^{β} denotes the equilibrium conflict profits of the sellers. By considering a 45-degree segment that connects Γ^{β} and $\mathcal{F}^{SP}$, we can identify the profits in case cartelization succeeds, C^{β}. Such a segment allows to equally share the surplus for a given conflict point. Notice that, the equilibrium expected profits of the sellers are convex combinations of the profits in case cartelization succeeds (C^{β}) and in case cartelization fails (Γ^{β}) with weights equal to β and $1-\beta$, respectively. Hence, the equilibrium expected profits, $(\hat{\pi}_M(\alpha,\beta), \hat{\pi}_D(\alpha,\beta))$, lie on the segment that connects Γ^{β} and C^{β}.

Figure 2 focuses on $\alpha = 0.9$ $\gamma = 100$ and considers different values of β. In the diagram for $\beta = 0$ (upper-left panel) the threat point Γ^0 is the non-cooperative equilibrium with M choosing the monopoly price and D optimally adjusting to it. If sellers would successfully cooperate, for which the probability is $\beta = 0$, they would share the maximal joint profits as indicated by C^0. In the diagram for $\beta = 0.5$ (upper-right panel) the threat point $\Gamma^{0.5}$ features lower threat profits for M and D what allows M to gain more at the cost of D in case of cartelization. For $\beta = 1$ (lower-left panel) the threat profits are even lower according to Γ^1, but occur with probability $1-\beta = 0$. So Γ^1 is the threat point underlying the Nash (1953)-bargaining solution C^1 in the profit space. Finally, the diagram for $\beta \in [-1, 1]$ (lower-right panel) shows that the equilibrium expected profits for M are increasing in β while the expected profits for D are at first decreasing and then increasing, reaching the maximum when $\beta = 1$.

Figure 3 replicates the same analysis but for $\gamma = 100$ and $\alpha = -0.9$. A negative externality ($\alpha < 0$) reduces the market profits and the possible gains from cooperation relative to a positive externality. From our results, we know that the qualitative pattern of the conflict and expected profits does not depend on α. Indeed, conflict profits (Γ^{β}) decrease for both sellers when β increases from 0 to 1. Moreover, the expected profits of M are increasing in β while the ones of D exhibit a u-shaped relationship reaching the maximum at $\beta = 1$.

5 Conclusions

As our example illustrates seller cooperation via exploiting customers is also possible when sellers do not compete, but are only related by positive or negative externalities. In our economy with one-sided dependency, this requires the monopolist M to undercut its price in case of a positive externality, respectively by increasing the price above the monopoly level in case of a negative externality.

Figure 2: Equilibrium expected profits for $\gamma = 100$, $\alpha = 0.9$ and different values of β.

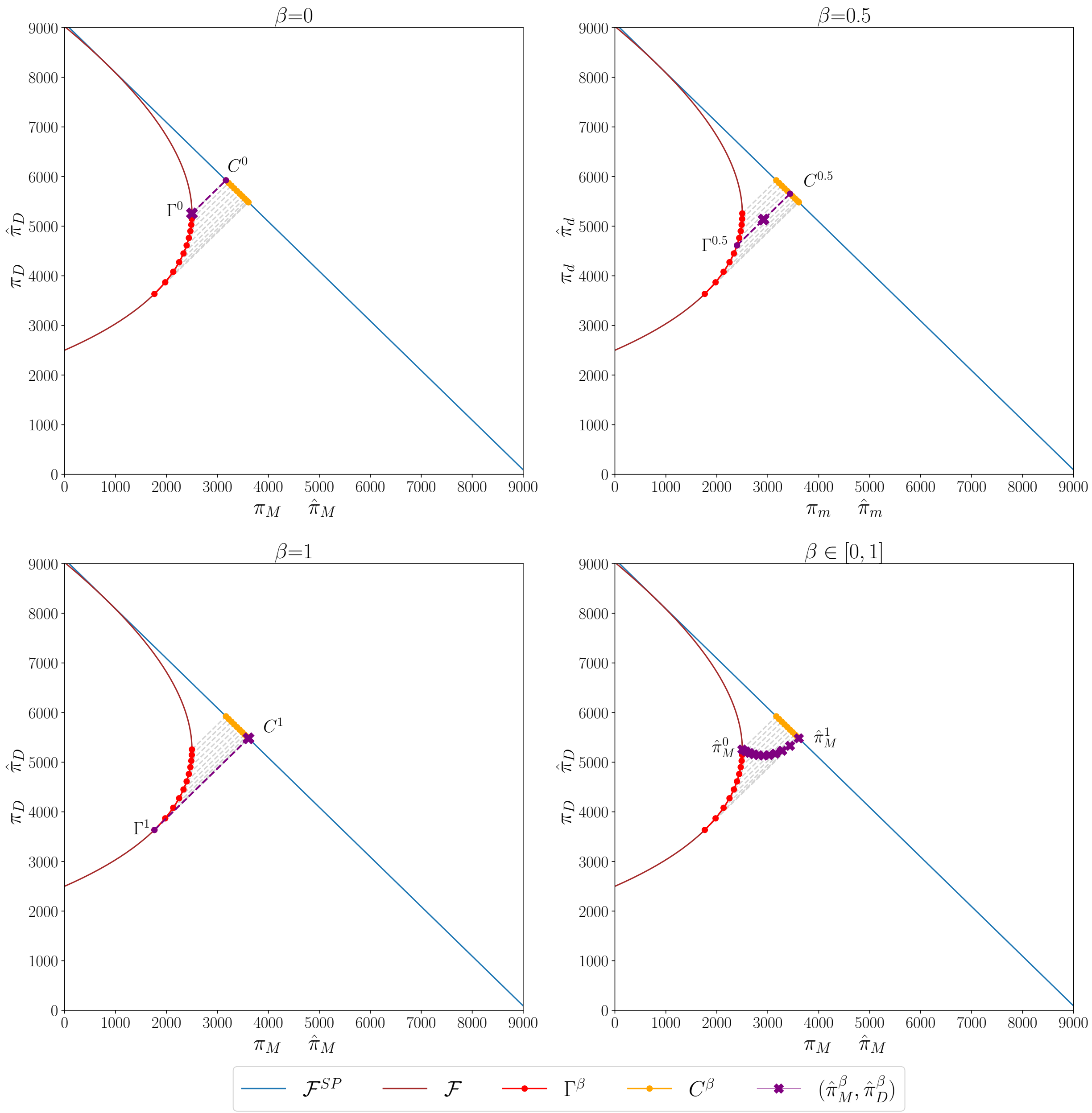
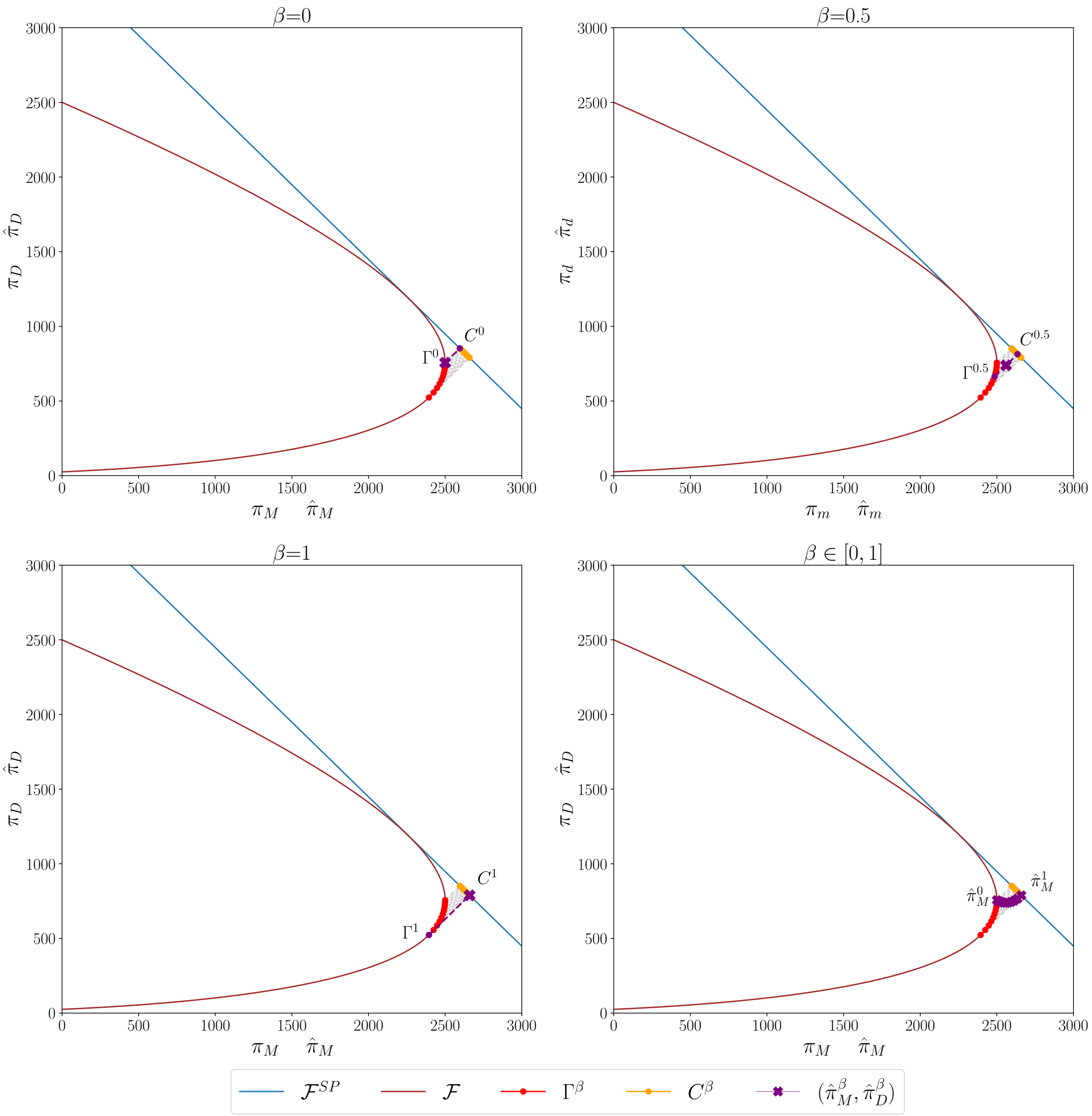


Figure 3: Equilibrium expected profits for $\gamma = 100$, $\alpha = -0.9$ and different values of β.



Taking the probability of successful cartelization as exogenously given, we have shown that increasing such probability does not make both sellers better off. This occurs for the monopolist M, but not always for the dependent seller: D would be only interested in sufficiently large β increases.

Even though in our economy the probability of successful cartelization is exogenously given, one can consider that such a probability is linked to the effort needed to build a structure to guarantee an effective communication and coordination among the members of the cartel. The relevance of relying on an effective communication and organization system has been argued both by theoretical papers and documented by empirical analysis. Our model predicts that when the probability of cartelization is low enough then the expected profits of the dependent seller are lower than the equilibrium non-cooperative ones. In this case the participation constraint of the dependent will be violated. Therefore, he will have no incentive to put any effort to sustain a collusive outcome. Alternatively, the dependent seller can boycott the attempt to establish any system of coordination among sellers. Furthermore, in case communication is necessary to support collusion (as for instance in Aubert et al., 2006) the dependent firm can whistle-blow any attempt to promote cartelization by the monopolist. In this respect, our results are in line with empirical findings of, among the others, Besley et al. (2021) and Block et al. (1981).

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A. Appendix to Section 2

A.1 Derivation of Equation (2)

To maximize the joint profits $\Sigma(p_M, p_D) = \pi_M(p_M) + \pi_D(p_M, p_D)$, we have to solve

$$\max \quad \Sigma(p_M, p_D), \quad \text{s.t.} \quad \gamma - p_M \geqslant 0, \quad \gamma + \alpha p_M - p_D \geqslant 0, \quad p_M, p_D \geqslant 0. \tag{A.1.1}$$

Recall that $\pi_M(p_M) = p_M(\gamma - p_M)$ and $\pi_D = p_D(\gamma + \alpha p_M - p_D)$. Let $\Sigma_M(p_M, p_D) = \gamma - 2p_M + \alpha p_D$ and $\Sigma_D(p_M, p_D) = \gamma - 2p_D + \alpha p_M$; furthermore, let μ_M and μ_D be the multiplier associated to the first and second constraint, respectively. The Kuhn-Tucker conditons are

$$\Sigma_M(p_M, p_D) \leqslant \mu_M - \alpha\mu_D, \quad \text{with equality if} \quad p_M > 0 \tag{A.1.2}$$

$$\Sigma_D(p_M, p_D) \leqslant \mu_D, \quad \text{with equality if} \quad p_D > 0 \tag{A.1.3}$$

$$\mu_M(\gamma - p_M) = 0 \tag{A.1.4}$$

$$\mu_D(\gamma + \alpha p_M - p_D) = 0 \tag{A.1.5}$$

Since $\Sigma(p_M, p_D)$ is a concave function, because its Hessian matrix is negative semidefinite, conditions (A.1.2)–(A.1.5) are sufficient. To consider all the possible cases,[19] we consider three scenarios:
Scenario (i): $0 \leqslant p_M < \gamma$ and $0 \leqslant p_D < \gamma + \alpha p_M$
Scenario (ii): $p_M = \gamma$ and $0 \leqslant p_D \leqslant \gamma + \alpha\gamma$
Scenario (iii): $0 \leqslant p_M < \gamma$ and $p_D = \gamma + \alpha p_M$

In scenario (i), (A.1.4) and (A.1.5) imply $\mu_M = 0$ and $\mu_D = 0$, respectively. Therefore, (A.1.2) and (A.1.3) imply

$$\gamma - 2p_M + \alpha p_D \leqslant 0, \quad \text{with equality if} \quad p_M > 0 \tag{A.1.6}$$

$$\gamma - 2p_D + \alpha p_M \leqslant 0, \quad \text{with equality if} \quad p_D > 0 \tag{A.1.7}$$

Scenario (i) includes four cases:
Case (i.a): $p_M = 0$ and $p_D = 0$, which implies, via (A.1.6) or (A.1.7), $\gamma \leqslant 0$. However, this is not possible because $\gamma > 0$ holds by assumption.
Case (i.b): $0 < p_M < \gamma$ and $p_D = 0$, which implies $p_M = \gamma/2$ via (A.1.6), hence $\gamma + \alpha(\gamma/2) \leqslant 0$ via (A.1.7). However, this implies $1 + \alpha/2 \leqslant 0$, which is not possible since $-1 < \alpha < 1$ holds by assumption.
Case (i.c): $p_M = 0$ and $0 < p_D < \gamma$, which implies $p_D = \gamma/2$ via (A.1.7), hence $\gamma + \alpha(\gamma/2) \leqslant 0$ via (A.1.6). However, as, in the previous case, this not possible.
Case (i.d): $0 < p_M < \gamma$ and $0 < p_D < \gamma + \alpha p_M$, which implies, via (A.1.6) and (A.1.7), $\gamma - 2p_M + \alpha p_D = 0$ and $\gamma - 2p_D + \alpha p_M = 0$. Solving the system gives $p_M = p_D = \gamma/(2 - \alpha)$.

Recall that, in scenario (ii), $p_M = \gamma$ and $0 \leqslant p_D \leqslant \gamma + \alpha\gamma$ hold, so that (A.1.4) implies $\mu_M \geqslant 0$. This scenario includes three cases:
Case (ii.a): $p_M = \gamma$ and $p_D = \gamma + \alpha\gamma$, which implies, via (A.1.2) and (A.1.3),

$$\gamma - 2\gamma + \alpha(\gamma + \alpha\gamma) = \mu_M - \alpha\mu_D \tag{A.1.8}$$

$$\gamma - 2(\gamma + \alpha\gamma) + \alpha\gamma = \mu_D \tag{A.1.9}$$

[19] The variables p_M and p_D can assume either an interior value or one boundary value (out of two). Therefore, there are $3^2 = 9$ cases altogether.

Substituting μ_D from (A.1.9) into (A.1.8) gives $\gamma - 2\gamma + \alpha(\gamma + \alpha\gamma) + \alpha\,[\gamma - 2(\gamma + \alpha\gamma) + \alpha\gamma] = \mu_M$. Simplyfing the left side gives $-\gamma = \mu_M$, which, however, is not possible, since $\gamma > 0$ holds by assumption and, in this scenario, we have $\mu_M \geqslant 0$.

Case (ii.b): $p_M = \gamma$ and $p_D = 0$, which implies $\mu_D = 0$ via (A.1.5), hence $\gamma - 2\gamma = \mu_M$ via (A.1.2). However, as explained in the previous case, this is not possible.

Case (ii.c): $p_M = \gamma$ and $0 < p_D < \gamma + \alpha\gamma$, which implies $\mu_D = 0$ via (A.1.5), hence, via (A.1.2) and (A.1.3),

$$\gamma - 2\gamma + \alpha p_D = \mu_M \tag{A.1.10}$$

$$\gamma - 2p_D + \alpha\gamma = 0 \tag{A.1.11}$$

Substituting p_D from (A.1.10) into (A.1.11) gives

$$\alpha\left(\frac{\gamma + \alpha\gamma}{2}\right) - \gamma = \mu_M,$$

hence

$$\left(\frac{\gamma}{2}\right)\left(\alpha + \alpha^2 - 2\right) = \mu_M$$

Since $|\alpha| < 1$ by assumption, $\alpha^2 < 1$ holds, which implies $\alpha + \alpha^2 < \alpha + 1 < 2$ and, therefore, $\alpha + \alpha^2 - 2 < 0$. However, this is not possible, since in this scenario we have $\mu_M \geqslant 0$.

Recall that, in scenario (iii), $0 \leqslant p_M < \gamma$ and $p_D = \gamma + \alpha p_M$ hold, so that (A.1.4) implies $\mu_M = 0$ and (A.1.5) implies $\mu_D \geqslant 0$. This scenario includes two cases:

Case (iii.a): $p_M = 0$ and $p_D = \gamma$, which implies, via (A.1.3), $\gamma - 2\gamma = \mu_D$. However, by the usual argument, this is not possible.

Case (iii.b): $0 < p_M < \gamma$ and $p_D = \gamma + \alpha p_M$, which implies $\gamma - 2(\gamma + \alpha p_M) + \alpha p_M = -(\gamma + \alpha p_M) = \mu_D$ via (A.1.3). However, this is not possible since $-1 < \alpha < 1$ and $0 < p_M < \gamma$ imply $\gamma + \alpha p_M > 0$.

Summing up, the only admissible case is (i.d), which corresponds to interior values of both variables p_M and p_D. **Therefore one has:**

$$p_M^J = p_D^J = \gamma/(2 - \alpha). \tag{A.1.12}$$

A.2 Derivation of Equation (3)

Given the prices p_M^J and p_D^J the maximal joint profits are equal to:

$$\begin{aligned}
\Sigma^J &= \pi_M(p_M^J) + \pi_D(p_M^J, p_D^J) \\
&= \left(\gamma - \frac{\gamma}{2-\alpha}\right)\frac{\gamma}{2-\alpha} + \left(\gamma - \frac{\gamma}{2-\alpha} + \alpha\frac{\gamma}{2-\alpha}\right)\frac{\gamma}{2-\alpha} \\
&= \left(2\gamma - \frac{2}{2-\alpha}\gamma + \frac{\alpha}{2-\alpha}\gamma\right)\frac{\gamma}{2-\alpha} \\
&= \left(\frac{4-2\alpha-2+\alpha}{2-\alpha}\gamma\right)\frac{\gamma}{2-\alpha} \\
&= \frac{1}{2-\alpha}\gamma^2
\end{aligned}$$

B. Appendix to Section 3

B.1 Derivation of Equation (9)

Best response of Seller M

The monopolist M chooses p_M to maximize the expected profits

$$\hat{\pi}_M(p_M, p_D) = \left(1 - \frac{\beta}{2}\right) \pi_M(p_M) + \left(\frac{\beta}{2}\right) \left(\Sigma^J - \pi_D(p_M, p_D)\right).$$

Since Σ^J is constant, maximizing the above function is equivalent to maximizing

$$\left(1 - \frac{\beta}{2}\right) \pi_M(p_M) - \left(\frac{\beta}{2}\right) \pi_D(p_M, p_D) = \left(1 - \frac{\beta}{2}\right) \left(\gamma p_M - p_M^2\right) - \left(\frac{\beta}{2}\right) \left(\gamma p_D - p_D^2 + \alpha p_M p_D\right),$$

which, since p_D is taken as given, is equivalent to maximizing

$$\left(1 - \frac{\beta}{2}\right) \left(\gamma p_M - p_M^2\right) - \left(\frac{\beta}{2}\right) \alpha p_M p_D. \tag{B.1.1}$$

The above function can be rearranged as follows:

$$\begin{aligned}\left[\gamma - \frac{\beta}{2}(\gamma + \alpha p_D)\right] p_M - \left(1 - \frac{\beta}{2}\right) p_M^2 &= \left[\left(1 - \frac{\beta}{2}\right)\gamma - \frac{\beta}{2}\alpha p_D\right] p_M - \left(1 - \frac{\beta}{2}\right) p_M^2 \\ &= \left[\left(\frac{2-\beta}{2}\right)\gamma - \frac{\beta}{2}\alpha p_D\right] p_M - \left(\frac{2-\beta}{2}\right) p_M^2 \\ &= \left(\frac{2-\beta}{2}\right)\left\{\left[\gamma - \left(\frac{\beta}{2-\beta}\right)\alpha p_D\right] p_M - p_M^2\right\}.\end{aligned}$$

Therefore, letting $\hat{\beta} = \beta/(2-\beta)$, we see that maximizing (B.1.1) is equivalent to maximizing

$$\left(\gamma - \hat{\beta}\alpha p_D\right) p_M - p_M^2. \tag{B.1.2}$$

The problem of the monopolist M is, therefore, to choose p_M to solve

$$\max_{p_M} \quad \left(\gamma - \hat{\beta}\alpha p_D\right) p_M - p_M^2,$$

$$\text{s.t.} \quad p_M \geqslant 0, \quad \gamma - p_M \geqslant 0, \quad \gamma + \alpha p_M - p_D \geqslant 0.$$

In this problem, p_D is taken as given, i.e., it varies parametrically; therefore, the analysis of the problem should take into any arbitrary value of p_D. However, since we are ultimately interested in the equilibrium choices of M and D, we can, for sake of concreteness, limit ourselves, to the values of p_D which solve problem (B.1). By definition, these values satisfy the constraint $\gamma + \alpha p_M - p_D \geqslant 0$, which, therefore, can be disregarded.

Summing up, to derive the best response of the monopolist M we have to solve

$$\max_{p_M} \quad \left(\gamma - \hat{\beta}\alpha p_D\right) p_M - p_M^2 \quad \text{s.t.} \quad p_M \geqslant 0, \quad \gamma - p_M \geqslant 0. \tag{B.1.3}$$

It is straightforward to verify that the objective function is concave; furthermore, it is easy to verify that $\gamma - \hat{\beta}\alpha p_D > 0$ whenever $\alpha \in (-1, 1)$. If $\alpha < 0$, this follows from the fact that all the other parameters are positive. If $\alpha > 0$, the restriction $\alpha < 1$ implies $(1+\alpha)/2 < 1$, and therefore,

$\gamma(1+\alpha)/2 < \gamma$. For the values of p_D given by (B.1.5) , $0 < p_D \leqslant \gamma(1+\alpha)/2$ will hold. Furthermore, since $\hat{\beta} \leqslant 1$ implies $\hat{\beta}\alpha < 1$, it is true that

$$\hat{\beta}\alpha p_D < \frac{\gamma(1+\alpha)}{2} < \gamma,$$

hence $(\gamma - \hat{\beta}\alpha p_D) > 0$, as claimed. This implies that $p_M = 0$ cannot be a solution to BR_M for our parameter constellations and the constraint $p_M \geqslant 0$ will never be binding; therefore, this constraint and its associated multiplier can be disregarded. Let $\lambda_M \geqslant 0$ denote the multiplier associated to the constraint $\gamma - p_M \geqslant 0$, so that the Kuhn-Tucker sufficient conditions are

$$(\gamma - \hat{\beta}\alpha p_D) - 2p_M = \lambda_M \quad \text{and} \quad \lambda_M(\gamma - p_M) = 0.$$

The boundary solution $p_M = \gamma$ would imply $-\hat{\beta}\alpha p_D - \gamma = \lambda_M \geq 0$. When considering $\alpha \in (0,1)$ we have $-\hat{\beta}\alpha p_D - \gamma < 0$ which contradicts the fact that $\lambda_M \geq 0$. When $-1 < \alpha \leq 0$ we can consider the values of p_D given by (B.1.5), $0 < p_D \leq \gamma(1+\alpha)/2$. Hence, since $\alpha < 0$ then $\hat{\beta}\alpha\gamma(1+\alpha)/2 < \gamma$. Therefore, $-\hat{\beta}\alpha p_D - \gamma < 0$ which contradicts the fact that $\lambda_M \geq 0$. From the above analysis, we can conclude that the solution to BR_M implies $\lambda_M = 0$ and

$$\hat{p}_M(p_D) = \frac{\gamma - \hat{\beta}\alpha p_D}{2} = \frac{\gamma}{2} - \frac{\beta\alpha p_D}{2(2-\beta)}. \tag{B.1.4}$$

Best response of Seller D

The dependent seller D chooses p_D to maximize the expected profits

$$\hat{\pi}_D(p_M, p_D) = \left(1 - \frac{\beta}{2}\right)\pi_D(p_M, p_D) + \left(\frac{\beta}{2}\right)\left(\Sigma^J - \pi_M(p_M)\right)$$

Since Σ^J is constant, $\pi_M(p_M)$ does not depend on p_D, and $1-\beta/2 > 0$, maximizing the above function is equivalent to maximizing the (non-cooperative) profits $\pi_D(p_M, p_D)$. Therefore, taking p_M as given, the dependent firm D chooses p_D to maximize the concave function $\pi_D(p_M, p_D) = (\gamma + \alpha p_M)\, p_D - p_D^2$. To derive the best response $p_D(p_M)$ we have to solve

$$\max_{p_D} \quad (\gamma + \alpha p_M)\, p_D - p_D^2 \quad \text{s.t.} \quad p_D \geqslant 0, \quad \gamma + \alpha p_M - p_D \geqslant 0.$$

Since we are ultimately interested in the equilibrium choices of M and D, we can, for sake of concreteness, assume that p_M satisfies $0 \leqslant p_M \leqslant \gamma$, since these restrictions will be imposed in deriving the best response of the monopolist firm M. Together with the assumption $-1 < \alpha < 1$, this implies $\gamma + \alpha p_M > 0$. Therefore, $p_D = 0$ cannot be a solution to BR_D for our parameter constellations and the constraint $p_D \geqslant 0$ will never be binding; therefore, this constraint and its associated multiplier can be disregarded. Let $\lambda_D \geqslant 0$ denote the multiplier associated to the constraint $\gamma + \alpha p_M - p_D \geqslant 0$, so that the Kuhn-Tucker sufficient conditions are

$$(\gamma + \alpha p_M) - 2p_D = \lambda_D \quad \text{and} \quad \lambda_D(\gamma + \alpha p_M - p_D) = 0.$$

The boundary solution $p_D = \gamma + \alpha p_M$ would imply $(\gamma + \alpha p_M)/2 = (\gamma + \alpha p_M) + \lambda_D/2$, hence

$$\frac{\gamma + \alpha p_M}{2} \geqslant \gamma + \alpha p_M,$$

which is not possible. Therefore, $p_D = \gamma + \alpha p_M$ cannot be a solution to BR_D for our parameters constellation. From the above analysis, we can conclude that the solution to BR_D implies $\lambda_D = 0$ and

$$\hat{p}_D(p_M) = \frac{\gamma}{2} + \frac{\alpha p_M}{2}. \tag{B.1.5}$$

B.2 Derivation of Equation (10)

Given the best responses:

$$\hat{p}_M(p_D) = \frac{\gamma}{2} - \frac{\alpha\beta p_D}{2(2-\beta)} \quad \text{and} \quad \hat{p}_D(p_M) = \frac{\gamma}{2} + \frac{\alpha p_M}{2},$$

we can substitute $\hat{p}_D$ in the first equation and get:

$$p_M(\alpha,\beta) = \frac{\gamma}{2} - \frac{\alpha\beta\left(\frac{\gamma}{2} + \frac{\alpha p_M(\alpha,\beta)}{2}\right)}{2(2-\beta)} \Leftrightarrow p_M(\alpha,\beta) = \frac{\gamma}{2} - \frac{\alpha\beta\gamma}{4(2-\beta)} - \frac{\alpha^2\beta p_M(\alpha,\beta)}{4(2-\beta)}$$

Hence,

$$p_M(\alpha,\beta) = \gamma\left(\frac{2(2-\beta)-\alpha\beta}{4(2-\beta)}\right) - \frac{\alpha^2\beta p_M(\alpha,\beta)}{4(2-\beta)} \quad = \left(\frac{4(2-\beta)+\alpha^2\beta}{4(2-\beta)}\right) = \gamma\left(\frac{2(2-\beta)-\alpha\beta}{4(2-\beta)}\right)$$

$$\Leftrightarrow p_M(\alpha,\beta) = \gamma\left(\frac{2(2-\beta)-\alpha\beta}{4(2-\beta)+\alpha^2\beta}\right) \Leftrightarrow p_M(\alpha,\beta) = \gamma\left(\frac{4-(2+\alpha)\beta}{8-(4-\alpha^2)\beta}\right) \tag{B.2.1}$$

By substituting $p_M(\alpha,\beta)$ in $\hat{p}_D(p_M)$ we have:

$$\begin{aligned}
p_D(\alpha,\beta) &= \frac{\gamma}{2} + \alpha\frac{\gamma\left(\frac{4-(2+\alpha)\beta}{8-(4-\alpha^2)\beta}\right)}{2} \\
&= \gamma\frac{8-(4-\alpha^2)\beta + 4\alpha - (2+\alpha)\alpha\beta]}{2[8-(4-\alpha^2)\beta]} \\
&= \gamma\frac{8+4\alpha-(2\alpha+\alpha^2+4-\alpha^2)\beta}{2[8-(4-\alpha^2)\beta]} \\
&= 2\gamma\frac{2(2+\alpha)-(2+\alpha)\beta}{2[8-(4-\alpha^2)\beta]} \\
&= \gamma\frac{(2+\alpha)(2-\beta)}{8-(4-\alpha^2)\beta}
\end{aligned} \tag{B.2.2}$$

B.3 Derivation of Equation (11)

Let us consider the difference between the two equilibrium prices:

$$
\begin{aligned}
p_M(\alpha,\beta) - p_D(\alpha,\beta) &= \frac{\gamma\,[4-(2+\alpha)\beta]}{8-(4-\alpha^2)\beta} - \frac{\gamma(2+\alpha)(2-\beta)}{8-(4-\alpha^2)\beta} \\
&= \frac{\gamma\,[4-(2+\alpha)\beta]}{8-(4-\alpha^2)\beta} - \frac{\gamma[4+2\alpha-(2+\alpha)\beta]}{8-(4-\alpha^2)\beta} \\
&= -\frac{2\alpha}{8-(4-\alpha^2)\beta} \qquad \text{(B.3.1)}
\end{aligned}
$$

B.4 Derivation of Equation (13) and Equation (14)

Derivation Equation (13)

Regarding $p_M(\alpha, \beta)$ one has:

$$\frac{\partial p_M(\alpha,\beta)}{\partial \beta} = \gamma \frac{-(2+\alpha)[8-(4-\alpha^2)\beta] + (4-\alpha^2)[4-(2+\alpha)\beta]}{[8-(4-\alpha^2)\beta]^2}$$

$$= \gamma \frac{-8(2+\alpha) + (4-\alpha^2)(2+\alpha)\beta + 4(4-\alpha^2) - (4-\alpha^2)(2+\alpha)\beta}{[8-(4-\alpha^2)\beta]^2}$$

$$= \frac{-4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2} \tag{B.4.1}$$

Hence, $\partial p_M(\alpha,\beta)/\partial\beta$ depends on α, it is negative when $\alpha > 0$ and positive when $\alpha < 0$.

Derivation Equation (14)

Regarding $p_D(\alpha, \beta)$ one has:

$$\frac{\partial p_D(\alpha,\beta)}{\partial \beta} = \frac{\partial p_M^\beta}{\partial \beta} + \frac{2\alpha(4-\alpha^2)}{[8-(4-\alpha^2)\beta]^2}\gamma$$

$$= \frac{4(-2\alpha-\alpha^2) + (4-\alpha^2)2\alpha}{[8-(4-\alpha^2)\beta]^2}\gamma$$

$$= \frac{-2\alpha^2(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\gamma \tag{B.4.2}$$

Hence, p_D is decreasing in β regardless the sign of α.

C. Appendix to Section 4

C.1 Derivation of Equation (15)

To save on notation, in this Appendix we write $\pi_M(\alpha,\beta)$ for $\pi_M(p_M(\alpha,\beta))$ and $\pi_D(\alpha,\beta)$ for $\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))$.

To compute $S(\alpha,\beta)$, we first compute $\pi_M(\alpha,\beta)$ and $\pi_D(\alpha,\beta)$ using $p_M(\alpha,\beta)$ and $p_M(\alpha,\beta)$. As for $\pi_M(\alpha,\beta)$, one has:

$$\begin{aligned}
\pi_M(\alpha,\beta) &= [\gamma - p_M(\alpha,\beta)]p_M(\alpha,\beta) \\
&= \left(\gamma - \gamma\frac{4-(2+\alpha)\beta}{[8-(4-\alpha^2)\beta]}\right)\gamma\frac{4-(2+\alpha)\beta}{[8-(4-\alpha^2)\beta]} \\
&= \frac{\gamma^2[4-(2+\alpha)\beta]}{[8-(4-\alpha^2)\beta]^2}\left(8-(4-\alpha^2)\beta-4+(2+\alpha)\beta\right) \\
&= \frac{\gamma^2[4-(2+\alpha)\beta]}{[8-(4-\alpha^2)\beta]^2}\left(4-\beta(2-\alpha-\alpha^2)\right) \\
&= \gamma^2\frac{16-4\beta(4-\alpha^2)+\beta^2(4-3\alpha^2-\alpha^3)}{[8-(4-\alpha^2)\beta]^2}
\end{aligned}$$

As for $\pi_D(\alpha,\beta)$, one has:

$$\begin{aligned}
\pi_D(\alpha,\beta) &= [\gamma - p_D(\alpha,\beta)+\alpha p_M(\alpha,\beta)]p_D(\alpha,\beta) \\
&= \left[\gamma - \gamma\frac{(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]^2} + \alpha\gamma\frac{4-(2+\alpha)\beta}{[8-(4-\alpha^2)\beta]}\right]\gamma\frac{(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]} \\
&= \gamma^2\frac{(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]^2}\left[8-(2-\alpha)(2+\alpha)\beta-(2+\alpha)(2-\beta)+4\alpha-\alpha(2+\alpha)\beta\right] \\
&= \gamma^2\frac{(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]^2}\left[8-(2-\alpha)(2+\alpha)\beta-(2-\alpha)(2-\beta)+4\alpha-\alpha(2+\alpha)\beta\right] \\
&= \gamma^2\frac{(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]^2}\left[4(2+\alpha)-2(2+\alpha)\beta-(2-\alpha)(2-\beta)\right] \\
&= \gamma^2\frac{[(2+\alpha)(2-\beta)]^2}{[8-(4-\alpha^2)\beta]^2} \\
&= \gamma^2\frac{4(4+4\alpha+\alpha^2)-4\beta(4+4\alpha+\alpha^2)+\beta^2(4+4\alpha+\alpha^2)}{[8-(4-\alpha^2)\beta]^2}
\end{aligned}$$

Hence one has:

$$\pi_M(\alpha,\beta)+\pi_D(\alpha,\beta)=\gamma^2\frac{[4(8+4\alpha+\alpha^2)-4\beta(8+4\alpha)+\beta^2(8+4\alpha-2\alpha^2-\alpha^3)]}{[8-(4-\alpha^2)\beta]^2} \tag{C.1.1}$$

We can now compute the surplus as follows:

$$\begin{aligned}
S(\alpha,\beta) &= \frac{\gamma^2}{2-\alpha}-\pi_M(\alpha,\beta)-\pi_D(\alpha,\beta)\\
&= \frac{\gamma^2[8-(4-\alpha^2)\beta]^2}{(2-\alpha)[8-(4-\alpha^2)\beta]^2}-\pi_M(\alpha,\beta)-\pi_D(\alpha,\beta)\\
&= \frac{\gamma^2[64-16(4-a^2)\beta+(4-a^2)^2\beta^2]}{(2-\alpha)([8-(4-\alpha^2)\beta]^2)}\\
&-\gamma^2\frac{(2-\alpha)\left[4(8+4\alpha+\alpha^2)-16\beta(2+\alpha)+\beta^2(8+4\alpha-2\alpha^2-\alpha^3)\right]}{(2-\alpha)[8-(4-\alpha^2)\beta]^2}\\
&= \frac{\gamma^2[64-16(4-a^2)\beta+(4-a^2)^2\beta^2]}{(2-\alpha)([8-(4-\alpha^2)\beta]^2)}\\
&-\gamma^2\frac{[4(16-2\alpha^2-\alpha^3)-16\beta(4-\alpha^2)+(2-\alpha)\beta^2(8+4\alpha-2\alpha^2-\alpha^3)]}{(2-\alpha)[8-(4-\alpha^2)\beta]^2}\\
&= \frac{\gamma^2[4(2\alpha^2+\alpha^3)+(2-a)\beta^2((4-\alpha^2)(2+\alpha)-8-4\alpha+2\alpha^2+\alpha^3)]}{(2-\alpha)([8-(4-\alpha^2)\beta]^2)}\\
&= \frac{\gamma^2[4\alpha^2(2+\alpha)+(2-a)\beta^2((8+4\alpha-2\alpha^2-\alpha^3)-8-4\alpha+2\alpha^2+\alpha^3)]}{(2-\alpha)([8-(4-\alpha^2)\beta]^2)}\\
&= \frac{(2+\alpha)(4\alpha^2\gamma^2)}{(2-a)[8-(4-\alpha^2)\beta]^2}.
\end{aligned}$$

Therefore it is immediate to verify that $S(\alpha,\beta)>0$ for evey $\alpha\in(-1,1)$, with $\alpha\neq 0$.

C.2 Derivation of Equation (17)

Let us compute $\partial \pi_M^*(\alpha,\beta)/\partial\beta$:

$$\frac{\partial \pi_M^*(\alpha,\beta)}{\partial\beta} = \frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} + \left(\frac{1}{2}\right) S(\alpha,\beta) + \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta}. \tag{C.2.1}$$

By direct computation, one has:

$$\begin{aligned}\frac{\partial S(\alpha,\beta)}{\partial\beta} = &-\frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta}\\ &-\frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} - \frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_D}\frac{\partial p_D(\alpha,\beta)}{\partial\beta}.\end{aligned}$$

From the profit maximization of seller D it follows that:

$$\frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_D} = 0,$$

and therefore one has:

$$\frac{\partial S(\alpha,\beta)}{\partial\beta} = -\frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} - \frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta}, \tag{C.2.2}$$

hence:

$$\begin{aligned}\frac{\partial \pi_M^*(\alpha,\beta)}{\partial\beta} = &\left(\frac{1}{2}\right) S(\alpha,\beta)\\ &+\left[\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial p_M} - \left(\frac{\beta}{2}\right)\frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\right]\frac{\partial p_M(\alpha,\beta)}{\partial\beta}.\end{aligned} \tag{C.2.3}$$

From the profit maximization of seller M it follows that:

$$\left(1-\frac{\beta}{2}\right)\frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial p_M} - \left(\frac{\beta}{2}\right)\frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M} = 0, \tag{C.2.4}$$

so that finally one has:

$$\frac{\partial \pi_M^*(\alpha,\beta)}{\partial\beta} = \left(\frac{1}{2}\right) S(\alpha,\beta). \tag{C.2.5}$$

C.3 Derivation of Equation (18)

Let us compute $\partial\hat{\pi}^*_D(\alpha,\beta)/\partial\beta$:

$$\frac{\partial\hat{\pi}^*_D(\alpha,\beta)}{\partial\beta} = \frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} + \frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_D}\frac{\partial p_D(\alpha,\beta)}{\partial\beta}$$

$$+\left(\frac{1}{2}\right)S(\alpha,\beta) + \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta}.$$

From the profit maximization of seller D it follows that:

$$\frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_D} = 0.$$

Therefore, using (C.7.1) one has:

$$\frac{\partial\pi^*_D(\alpha,\beta)}{\partial\beta} = \left(\frac{1}{2}\right)S(\alpha,\beta)$$

$$+\left[\left(1-\frac{\beta}{2}\right)\frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M} - \left(\frac{\beta}{2}\right)\frac{\partial\pi_M(p_M(\alpha,\beta))}{\partial p_M}\right]\frac{\partial p_M(\alpha,\beta)}{\partial\beta}.$$

The crucial observation is that, differently from what happens for (C.2.3), one cannot claim that the term in the square brackets is zero, since in this case the envelope argument does not apply. However, from (C.2.4) one has:

$$\left(\frac{\beta}{2}\right)\frac{\partial\pi_M(p_M(\alpha,\beta))}{\partial p_M} - \left(\frac{\beta}{2}\right)\frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M} = -\frac{\partial\pi_M(p_M(\alpha,\beta))}{\partial p_M}.$$

so that one has:

$$\frac{\partial\pi^*_D(\alpha,\beta)}{\partial\beta} = \left(\frac{1}{2}\right)S(\alpha,\beta) + \left[\frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M} - \frac{\partial\pi_M(p_M(\alpha,\beta))}{\partial p_M}\right]\frac{\partial p_M(\alpha,\beta)}{\partial\beta}.$$

Morevover, from (C.2.1) and (C.2.5) it follows that:

$$-\frac{\partial\pi_M(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} = \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta}.$$

Furthermore, from (C.7.1) one also has:

$$\frac{\partial\pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} = -\frac{\partial\pi_M(p_M(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial\beta} - \frac{\partial S(\alpha,\beta)}{\partial\beta}$$

$$= \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta} - \frac{\partial S(\alpha,\beta)}{\partial\beta}.$$

Therefore finally one has:

$$\frac{\partial\pi^*_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial\beta} = \left(\frac{1}{2}\right)S(\alpha,\beta) + \left[\left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta} - \frac{\partial S(\alpha,\beta)}{\partial\beta} + \left(\frac{\beta}{2}\right)\frac{\partial S(\alpha,\beta)}{\partial\beta}\right]$$

$$= \left(\frac{1}{2}\right)S(\alpha,\beta) - (1-\beta)\frac{\partial S(\alpha,\beta)}{\partial\beta}$$

C.4 Derivation of Equation (19)

By substituting the equilibrium prices $p_M(\alpha, \beta)$ and $p_D(\alpha, \beta)$ in (C.7.1), one has:

$$\frac{\partial S(\alpha,\beta)}{\partial \beta} = -\left[\gamma - 2\frac{\gamma\left[4-(2+\alpha)\beta\right]}{8-(4-\alpha^2)\beta} + \alpha\frac{\gamma(2+\alpha)(2-\beta)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

$$= -\gamma\left[\frac{8-(4-\alpha^2)\beta - 8 + 2(2+\alpha)\beta + \alpha(2+\alpha)(2-\beta)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

$$= -\gamma\left[\frac{-(4-\alpha^2)\beta + (2+\alpha)\beta(2-\alpha) + 2\alpha(2+\alpha)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

$$= -\gamma\left[\frac{-(2-\alpha)(2+\alpha)\beta + (2+\alpha)\beta(2-\alpha) + 2\alpha(2+\alpha)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

$$= -2\gamma\left[\frac{\alpha(2+\alpha)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

Recalling the result in **??**, one verifies that $-\partial p_M(\alpha,\beta)/\partial\beta$ has the same sign as α, and so does the term in the square brakets. It follows that $\partial S(\alpha,\beta)/\partial\beta > 0$ for any $\alpha \in (-1, 1)$, with $\alpha \neq 0$.

C.5 Proof of Proposition 1

Using the results from Appendix C.1 and Appendix C.4, one has:

$$\frac{\partial \hat{\pi}_D^*(\alpha,\beta)}{\partial \beta} = \left(\frac{1}{2}\right) S(\alpha,\beta) - (1-\beta)\frac{\partial S(\alpha,\beta)}{\partial \beta}$$

$$= \left(\frac{1}{2}\right) \frac{(2+\alpha)(4\alpha^2\gamma^2)}{(2-\alpha)[8-(4-\alpha^2)\beta]^2} - (1-\beta)\left[\frac{-2\gamma\alpha(2+\alpha)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

Furthermore, by direct computeation one has:

$$\frac{\partial p_M(\alpha,\beta)}{\partial \beta} = \gamma\frac{-(2+\alpha)[8-(4-\alpha^2)\beta]+(4-\alpha^2)[4-(2+\alpha)\beta]}{[8-(4-\alpha^2)\beta]^2}$$

$$= \gamma\frac{-8(2+\alpha)+(4-\alpha^2)(2+\alpha)\beta+4(4-\alpha^2)-(4-\alpha^2)(2+\alpha)\beta}{[8-(4-\alpha^2)\beta]^2}$$

$$= \frac{-4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}$$

Therefore one has:

$$\frac{\partial \pi_D^*(\alpha,\beta)}{\partial \beta} = \left(\frac{1}{2}\right) \frac{(2+\alpha)(4\alpha^2\gamma^2)}{(2-a)[8-(4-\alpha^2)\beta]^2} - (1-\beta)\left[\frac{-2\alpha\gamma(2+\alpha)}{8-(4-\alpha^2)\beta}\right]\frac{-4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}$$

$$= \frac{2\alpha^2\gamma^2(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\left[\frac{8-(4-\alpha^2)\beta-4(1-\beta)(2+\alpha)(2-\alpha)}{(2-\alpha)[8-(4-\alpha^2)\beta]}\right]$$

$$= \frac{2\alpha^2\gamma^2(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\left[\frac{8-(4-\alpha^2)\beta-4(1-\beta)(4-\alpha^2)}{(2-\alpha)[8-(4-\alpha^2)\beta]}\right]$$

$$= \frac{2\alpha^2\gamma^2(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\left[\frac{8-(4-\alpha^2)\beta-4(4-\alpha^2)+4\beta 4-\alpha^2)}{(2-\alpha)[8-(4-\alpha^2)\beta]}\right]$$

$$= \frac{2\alpha^2\gamma^2(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\left[\frac{-4(2-\alpha^2)+3\beta(4-\alpha^2)}{(2-\alpha)[8-(4-\alpha^2)\beta]}\right]$$

The derivative is positive (negative) if the numerator of the term in square brackets is positive (negative). By focusing on this term, one has:

$$-4(2-\alpha^2)+3\beta(4-\alpha^2) \geq 0 \Leftrightarrow \beta \geq \frac{4(2-\alpha^2)}{3(4-\alpha^2)} = \bar{\beta}$$

C.6 Derivation of Equation (21)

The equilibrium expected profits for $\beta = 1$ are equal to:

$$\hat{\pi}_D^*(\alpha, 1) = \pi_D(p_M(\alpha, 1), p_D(\alpha, 1)) + \frac{1}{2}S(\alpha, 1)$$

$$= \frac{\gamma^2(2+\alpha)^2}{[8-(4-\alpha^2)]^2} + \frac{1}{2}\frac{4\alpha^2\gamma^2(2+\alpha)}{(2-\alpha)[8-(4-\alpha^2)]^2}$$

$$= \frac{\gamma^2(2+\alpha)}{[4+\alpha^2]^2}\left[\frac{(2-\alpha)(2+\alpha)+2\alpha^2}{(2-\alpha)}\right]$$

$$= \frac{\gamma^2(2+\alpha)(4+a^2)}{(4+\alpha^2)^2(2-\alpha)}$$

The non-cooperative profit of D is equal to:

$$\hat{\pi}_D^*(\alpha, 0) = \pi_D\left(p_M = \frac{\gamma}{2}, p_D = \frac{\gamma(2+\alpha)}{2}\right)$$

$$= \left(\gamma - \frac{1}{4}\gamma(2+\alpha) + \alpha\frac{\gamma}{2}\right)\frac{1}{4}\gamma(2+\alpha)$$

$$= \frac{1}{4}\gamma(4-(2+\alpha)+2\alpha)\frac{1}{4}\gamma(2+\alpha)$$

$$= \frac{1}{16}\gamma^2(2+\alpha)^2$$

By comparing the equilibrium profits when $\beta = 1$ and $\beta = 0$ we get:

$$\hat{\pi}_D^*(\alpha, 1) - \hat{\pi}_D^*(\alpha, 0) = \frac{\gamma^2(2+\alpha)(4+a^2)}{(2-\alpha)(4+\alpha^2)^2} - \frac{(2+\alpha)^2}{16}\gamma^2$$

$$= \gamma^2(2+\alpha)\left[\frac{(4+\alpha^2)}{(2-\alpha)(4+\alpha^2)^2} - \frac{(2+a)}{16}\right]$$

$$= \gamma^2(2+\alpha)\left[\frac{1}{(2-\alpha)(4+\alpha^2)} - \frac{(2+a)}{16}\right]$$

$$= \gamma^2(2+\alpha)\left[\frac{16-(4-\alpha^2)(4+\alpha^2)}{16(2-\alpha)(4+\alpha^2)}\right]$$

$$= \gamma^2(2+\alpha)\left[\frac{16-(16-\alpha^4)}{16(2-\alpha)(4+\alpha^2)}\right]$$

$$= \frac{\gamma^2(2+\alpha)}{16(2-\alpha)(4+\alpha^2)}\alpha^4 > 0$$

C.7 Equilibrium conflict profits as a function of β

For the derivative of the equilibrium prices with respect to β see Appendix B.4.

Seller M

Considering the market profits π_M,

$$\begin{aligned}
\frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial \beta} &= \frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial \beta} \\
&= [\gamma - 2p_M(\alpha,\beta)]\,\frac{\partial p_M^\beta}{\partial \beta} \\
&= \left[\gamma - 2\frac{\gamma[4-(2+\alpha)\beta]}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M^\beta}{\partial \beta} \\
&= \gamma\left[\frac{8-(4-\alpha^2)\beta - 8 + 2(2+\alpha)\beta}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M^\beta}{\partial \beta} \\
&= \gamma\left[\frac{8-(4-\alpha^2)\beta - 8 + 2(2+\alpha)\beta}{8-(4-\alpha^2)\beta}\right]\left[-\frac{4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\right] \\
&= \left[\frac{\alpha(2+\alpha)\beta\gamma}{8-(4-\alpha^2)\beta}\right]\left[-\frac{4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\right] \\
&= -\left[\frac{4\alpha^2\gamma^2(2+\alpha)^2\beta}{[8-(4-\alpha^2)\beta]^3}\right] \qquad \text{(C.7.1)}
\end{aligned}$$

which is negative regardless of α. Therefore, $\pi_M(\alpha,\beta)$ is decreasing regardless of α.

Seller D

Considering the market profits π_D, one has:

$$\frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial \beta} = \frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_M}\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

$$+ \frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial p_D}\frac{\partial p_D(\alpha,\beta)}{\partial \beta}$$

$$= [\alpha p_D(\alpha,\beta)]\frac{\partial p_M(\alpha,\beta)}{\partial \beta} + [\gamma - 2p_D(\alpha,\beta) + \alpha p_M(\alpha,\beta)]\frac{\partial p_D(\alpha,\beta)}{\partial \beta}$$

$$= \left[\frac{\alpha\gamma(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta}$$

$$+ \left[\gamma - \frac{2\gamma(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]} + \frac{\alpha\gamma[4-(2+\alpha)\beta]}{[8-(4-\alpha^2)\beta]}\right]\frac{\partial p_D(\alpha,\beta)}{\partial \beta}$$

$$= \left[\frac{\alpha\gamma(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]}\right]\left[-\frac{4\alpha\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\right]$$

$$+ \left[8-(4-\alpha^2)\beta - 4(2+\alpha) + 2(2+\alpha)\beta + 4\alpha - \alpha(2+\alpha)\beta\right]\frac{\partial p_D(\alpha,\beta)}{\partial \beta}$$

$$= -\left[\frac{4\alpha^2\gamma^2(2+\alpha)^2(2-\beta)}{[8-(4-\alpha^2)\beta]^3}\right]$$

$$+ [-(2-\alpha)(2+\alpha)\beta + (2-\alpha)(2+\alpha)\beta]\frac{\partial p_D(\alpha,\beta)}{\partial \beta}$$

$$= -\left[\frac{4\alpha^2\gamma^2(2+\alpha)^2(2-\beta)}{[8-(4-\alpha^2)\beta]^3}\right] \tag{C.7.2}$$

Therefore, this is negative regardless the sign of α.

Difference of the variations of the two conflict profits

The difference between the two derivative is:

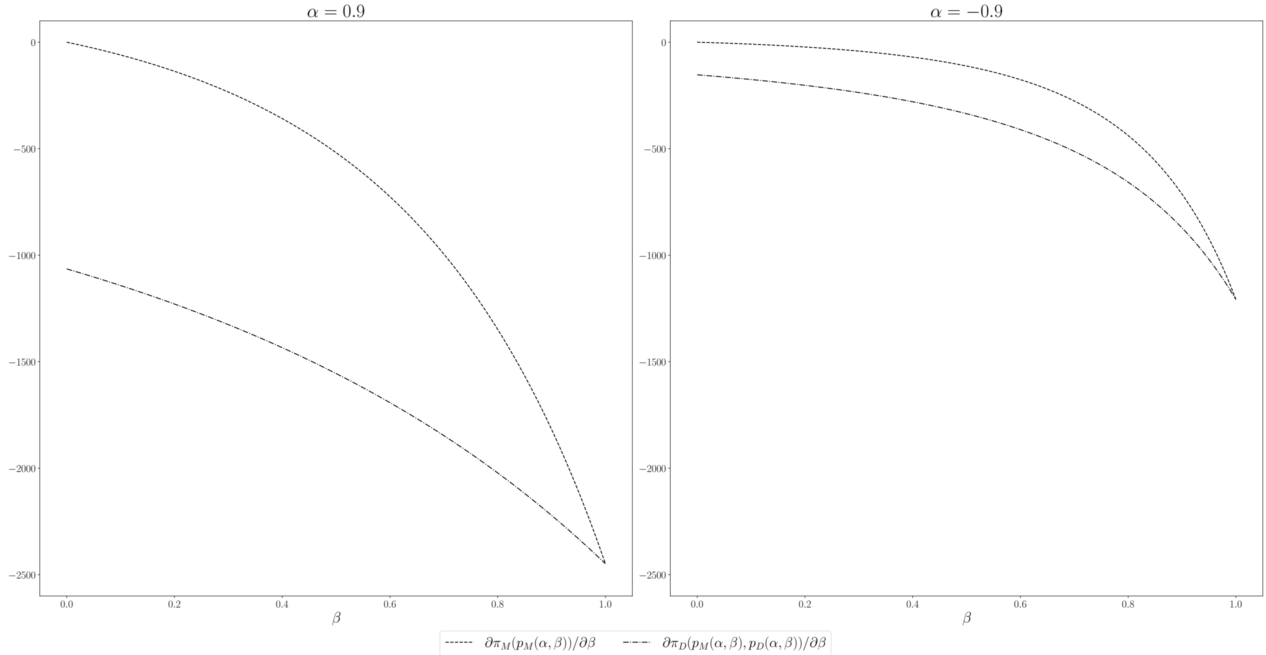
$$\frac{\partial \pi_M(p_M(\alpha,\beta))}{\partial \beta} - \frac{\partial \pi_D(p_M(\alpha,\beta),p_D(\alpha,\beta))}{\partial \beta} = \left[\frac{\alpha(2+\alpha)\beta\gamma}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M(\alpha,\beta)}{\partial \beta} - \left[\frac{\alpha\gamma(2+\alpha)(2-\beta)}{[8-(4-\alpha^2)\beta]}\right]\frac{\partial p_M}{\partial \beta}$$

$$= \left[\frac{\alpha(2+\alpha)\gamma\beta - \alpha\gamma(2+\alpha)(2-\beta)}{8-(4-\alpha^2)\beta}\right]\frac{\partial p_M}{\partial \beta}$$

$$= -\left[\frac{2\alpha(2+\alpha)\gamma(1-\beta)}{8-(4-\alpha^2)\beta}\right]\left[-\frac{2\alpha^2\gamma(2+\alpha)}{[8-(4-\alpha^2)\beta]^2}\right]$$

$$= -\left[\frac{8\alpha(2+\alpha)^2\gamma^2(1-\beta)}{[8-(4-\alpha^2)\beta]^3}\right] \tag{C.7.3}$$

Figure A.1: Variation of the equilibrium conflict profits as function of β



which is always negative regardless α. As shown in the figure below the difference is negative but increasing with β, meaning that the two values get closer as β approaches to 1.